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The stability of cracked rectangular plate with variable thickness using phase field method

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A R T I C L E I N F O

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A B S T R A C T

This study focuses on the investigation of the stability in a rectangular FGM plate with central crack. The plate thickness is changed linearly following the length of the plate. Using the Reissner-Mindlin first order shear deformation theory (FSDT), phase field theory and finite element method (FEM), the stability of fracture of the plate is determined. In order to ensure the reliability of the study, the obtained numerical results in this paper are compared with results reported in other publications. The work also presents the analysis of critical buckling computation for plate that have variation in thickness, the length of the crack on plate as well as the inclined angle of the crack. The numerical results show that the crack length impacts significantly to the critical buckling values of the plate, whereas the impact of inclined angle is less.

1. Introduction

The damage of structure is considered as the change of physical geometry of the structure comparing to the original state before using processes. Normally, the structural damages caused by defect are described by two parameters which are position and frustrated level. For instance, cracking, a form of typical damages in structure, is characterized by two parameters of position and its size. It is clearly that fracture mechanics as well as instability exploration have a really close relationship; with the development of numerous advanced structures nowadays, they get a great concentration from experts and researchers in engineering field. Among the instability problems, buckling behaviors is considered as one of the most serious phenomena, when the structure got high stress concentration or in-plane compressible forces due to the thermal or mechanical loads, then the structure could be made to be in buckling state before reaching to the yield stress state, there is no surprise if the capacity of load will be decreased greatly. There are several studies about buckling analysis in structures. Shariat and Eslami [1] has investigated the thermal buckling phenomenon of functionally graded plates (FGPs) by using a new closed-form numerical formula with the first order shear deformation theory to model the thick plate, while using the same theory, Golmakani [3] did the investigation of FGPs in circular deformation behaviors as applying thermal loading with temperature dependent properties, In a research of Najafizadeh and Haydari [2], they has applied the third order shear deformation theory to analyze the critical buckling temperature rise of a circular FGPs. Additionally, an analysis about static behavior and free vibration with the application of high order shear deformation theory combined finite element method (FEM) in FGMs plate has been analyzed by Talha and Singh [8]. On the other hand, Bilouei et al. [11] has analyzed the buckling behaviors of concrete column reinforced by nano-fiber reinforced polymer plate (NFRP), Kolahchi and Arani [30] was the firsts investigating the nonlinear buckling phenomenon of a conventional concrete column equipped single-walled carbon nanotubes (SWCNTs) resting on a ground, on another work [32], Kolahchi and Safari did a nonlinear dynamic stability exploration of embedded temperature dependent viscoelastic plates using the same reinforcement as (SWCNTs). Kolahchi [5] was also the first researcher who proposed the study in dynamic buckling analysis for sandwich nanoplates based on the visco-nonlocal-refined zigzag theory, then in another research [31], he did an analysis on the dynamic stability of an embedded piezoelectric nanoplate made of polyvinylidene fluoride (PVDF), or Shufrin et al. [7] figured out the buckling phenomenon of laminated plates subjected to in-plane shear forces, compression and tension loads. Duc [9] also proposed a study collection in nonlinear static and dynamic stability of functionally graded plates and shells. It is

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clearly that the buckling problem got many concentrations for many types of materials both in large deformation issues, and micro scale as well as nano scale [33–36].

In addition, fracture mechanic problem such cracking appearance have also been focusing for decades, since the basic problem was studied by Adams et al. [10] when an elastic bar with defect which decreases the local stiffness is modeled by an uniaxial spring and rebuilt the function in the purpose of determining the defect position according to the data from measurement of natural frequency, or Liang at al. [14] has explored successfully the general form of the frequent function for elastic beam with cracked which is described by a twisted spring, and the stiffness is computed according to the thickness of the cracks, Morassi [15] had set up the interfered functional model to analyze the natural frequency of the one-cracked beam when varying the stiffness. Narkis [12] had found the analytic solutions for crack position from measured data of two natural frequencies in the case of boundary condition as fully simple supported. It can be said that fractures have big impacts to the stability of structure during working process. Furthermore, improving the algorithm for treating cracks technically is also a difficult work to verify the numerical computation as closely to reality problem as possible, since crack formation is very complicated and hard to predict. Thanh [37] did a research work to develop a phantom-node method for shell elements for representing the crack which could have ability to treat arbitrary cracks independently of the mesh; in another study, Rabczuk [40] illustrated a mesh free method for dealing with the dynamic cracking problem in shell structure due to the high order continuity. Areias [38] proposed a crack propagation which could achieve the independence of mesh size, and the length scale that is compatible with the mesh, the analyzed algorithm could be applied in both quasi-brittle and ductile problems. Extended isogeometric element formulation (XIGA) is a well-known method for cracking numerical computation; Nguyen-Thanh [39] upgraded a XIGA in considering the crack problem which is through the thickness of thin shell structure, in this research, XIGA proposed a particular field near the crack tip and reproduced the discontinuities field across the crack.

With the development of numerical computation methods and computer technology, many useful methods have been applied to analyze structure behaviors with cracking. Some of the popular methods used to simulate and compute the cracking in structures could be named as extended finite element method (XFEM), smoothed finite element method (SFEM) and extended isogeometric analysis (XIGA). Dolbow et al. [13] has used extended finite element method (XFEM) to model the mix mode of the deflection according to the Mindlin-Reissner plate theory, Shaterzadeh [18] has studied the critical buckling temperature rise problem for a composite plate with a circular cutout by using FEM with the consideration of various factors as boundary conditions or gradient volume fraction index. Especially, among the studies of the impacts of cracking to structures, the analysis of linear plate buckling behavior with internal cracks by Baiz and Natarajan [17] got many focuses since they applied both XFEM and SFEM in the study, differently in the research of Tiantang Yu [16], he has carried out thermal buckling impacts by extended isogeometric analysis (XIGA) of FGPs with internal cracks.

Recently, phase-field theory was proposed as a new approach to simulate and compute numerically and effectively in static and dynamic defect problems such cracking since it shows a lot advantages when modeling and analyzing internal fracture in structure such as representing the geometries, shapes and the number of fractures more easily. By using phase-field method, Borden [19] has investigated the dynamic brittle fracture problem and proved that phase-field method greatly reduce the implementation complexity, on the other hand, Nguyen et al. also used phase-field to simulate crack initiation and propagation in 2D and 3D in heterogeneous material for the first time successfully [22]. The advantages of phase-field method have been showed in various researches, Amiri et al. [20] has used phase-field method in the exploration the fracture modeling for linear thin shells, while Areias et al. [23] used phase-field method in a study of finite-strain plates and shells analysis with element subdivision, Kuhn [21] proposed a continuum phase-field model for a various formulation of brittle fracture, furthermore, Ulmer et al. [27] have applied phase-field for fracture in both plate and shell in the situations with crack complex topologies, then handling succeed with diffusive crack modeling. Thom et al. [24]. have used this method to study the impact from the size and position of the cracks to the static instability of functionally graded material (FGM) plate by applying finite element method and a new third order shear deformation theory proposed by Shi [28], then in another study, he has analyzed the thermal buckling behavior of cracked FGM plate with considering the neutral surface since it is different totally to mid-surface in FGM plate by phase field method as well. Phase-field method was also applied in the study of Duc et al. [25,26] did the investigation of dynamic propagation of cracks FGM plate in damage mechanic problem. It can be said that phase-field method eventually get many focuses from scientists and experts, especially in fracture mechanics, and this method has advantages in numerous types of materials and shapes.

To the best of our knowledge, there has been recently no publication on the buckling of cracked plates with variable thickness.

In this paper, an analysis in critical buckling behavior of the cracked plate when the thickness is changed using phase-field method in order to consider the effect of internal cracks will be introduced; the model is built based on the first order shear deformation theory. The numerically computed results are taken in comparisons with the results of the references [4,6] to show the reliability of the study. Then, the impact of the change in thickness of the plate on critical buckling values in plate's instability is analyzed and discussed in the following sections. With the need of using and developing many complicated shape in plate material day by day, the achieved work is not only providing the analysis of buckling in plate with the changing of shape which is the thickness of plate in this study, as well as demonstrating and improving of phase field method in fracture computation, but also helping to predict the failure risks of buckling problem in engineering reality.

The second section will be presented the model based numerical computation as the first order shear deformation theory, then the next section will be crack modeling by applying phase-field method. The analyses for critical buckling numerical computations are discussed in the fourth section. Last but not least, some conclusions will be summarized on the conclusion section as the fifth.

2. Formulation for Reissner-Mindlin plate

Using the hypothesis of von Karman for the Reissner-Mindlin formulation, the 3D displacement field is expressed in term of middle plane as:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z\beta_x(x, y) \\ v(x, y, z) &= v_0(x, y) + z\beta_y(x, y) \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (1)$$

Where u, v, w are the displacements components in the x, y, z axes, respectively; β_x, β_y are the transverse normal rotations in the xz- and yz-planes; u_0, v_0, w_0 are displacement of middle surface.

From the expression of strains in 3D solid, the strains in terms of mid-plan deformation can be expressed as follow:

$$\begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\gamma} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\varepsilon}_p \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} z\boldsymbol{\varepsilon}_b \\ \boldsymbol{\gamma}_s \end{Bmatrix} \quad (2)$$

Where the vector $\boldsymbol{\varepsilon}_p$ and $\boldsymbol{\varepsilon}_b$ contain a membrane, bending and transverse strain, respectively.