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Constrained shell Finite Element Method for thin-walled members, Part 1: constraints for a single band of finite elements

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A B S T R A C T

In this paper a novel method for the analysis of thin-walled members is presented: the constrained finite element method. The method is basically a shell finite element analysis, but carefully defined constraints are applied which enforce the thin-walled member to deform in accordance with specific mechanical criteria, e.g., to force local, global or distortional deformations. The constrained finite element method is essentially similar to the constrained finite strip method, but the trigonometric longitudinal shape functions of the finite strip method are replaced by polynomial longitudinal shape functions, which – together with longitudinal discretization – transforms a finite strip into multiple finite elements. This change in longitudinal interpolation makes the method applicable for a wide range of practical problems not yet handled by other modal decomposition methods. The new shell finite element is briefly presented here, but the main focus of this paper is on how the constraining criteria can be applied for a thin-walled member. More specifically, in this paper a band of finite elements is discussed in detail, where 'band' is a segment of the member with multiple elements along the cross-section, but with one single finite element longitudinally. The possible base systems for the various deformation spaces are demonstrated here. Members built up from multiple bands are discussed and presented in a companion paper, where various numerical examples are also provided to illustrate the potential of the proposed constrained finite element method.

1. Introduction

A widely used practical approach to understand and analyse the complex behaviour of a structural member is to decompose the complex phenomenon into simpler ones, and then to interpret the complex phenomenon as a superposition of simpler phenomena. This is the reason why the deformations of a thin-walled beam or column member are frequently categorized into simpler yet practically meaningful deformation classes: global, distortional, local-plate and other classes, based on some characteristic features of the deformations.

In case of thin-walled members (e.g., cold-formed steel members) the deformations are frequently categorized into characteristic classes as follows: global (G), distortional (D), local-plate (L), shear (S) and transverse extension (T) behaviour. In many practical situations G, D and L are the most important ones.

The modal decomposition of the behaviour of a thin-walled member has been found especially useful to understand and analyse the stability behaviour, the behaviour which is governing in many practical situations due to the thin-walled nature, i.e., high slenderness of the structure. The classification is used in capacity prediction, too, and appears either implicitly or explicitly in current design standards for cold-formed steel, see [1, 2]. Though the knowledge of pure buckling modes and the values of the associated critical loads are essential in the design of thin-walled members, still there are practical cases when modal decomposition has not been possible. So far there have been two available methods with general modal decomposition features: the generalized beam theory (GBT), see e.g. [3–5] and the constrained finite strip method (cFSM), see e.g. [6–11]. Both methods are available in software applications, namely: GBTUL [12, 13] and CUFSM [14, 15], but either the method or its current implementation has limitations.

Many of cFSM limitations derive from FSM itself. FSM requires that the member is flat-walled and prismatic, as well as arbitrary end restraints cannot be handled efficiently. The requirement of prismatic sections, shared also by GBT, prevents direct application to tapered members, and members with holes. Recent works aimed at partially removing these limitations, by generalizing cFSM for certain end restraint [16, 17], or applying cFSM base functions or GBT cross-section deformation modes to modal identification of shell FEM deformations fields [18–22], or by applying the constraining technique for spline FSM [23] or shell FEM [24, 25], or by working out methods for the analysis of members with holes [26–34]. Nevertheless, all these attempts have not lead to a simple and easily applicable general method

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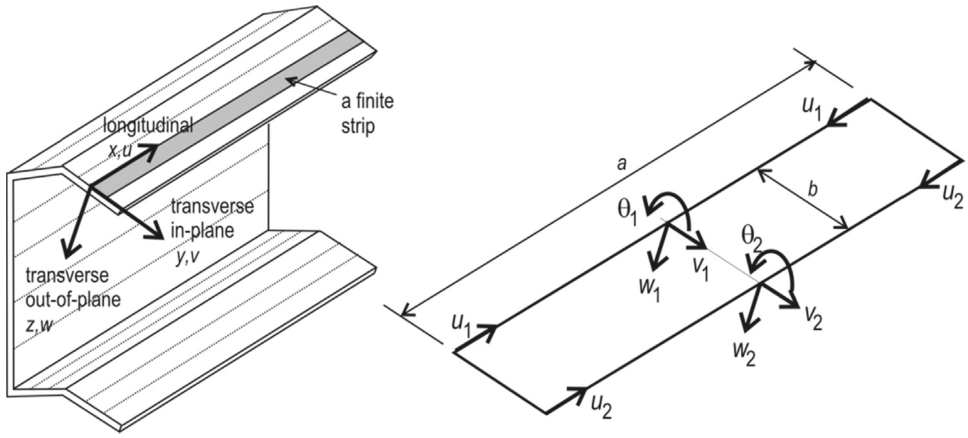

Fig. 1. Finite strip discretization, strip DOF, and notation.

with modal features.

In this paper a novel method is discussed. The proposed method follows the logic of cFSM, however, the longitudinal shape functions are changed and applied together with a longitudinal discretization. Thus, strips are transformed into multiple shell finite elements. The longitudinal shape functions are selected in such a way that modal decomposition similar to cFSM can be realized, therefore, the new method can be described as constrained finite element method (cFEM), possessing all the modal features of cFSM, but with significantly more flexibility in its application.

Though in [24, 25] constraints are applied to shell finite element models, the here-used cFEM is significantly different, since (a) cFEM provides a full modal decomposition, i.e., the whole displacement field is transformed into a modal system, which allows modal identification, too, (b) in cFEM the mechanical criteria of the modes are exactly satisfied, and (c) in cFEM adding constraints reduces the DOF number of the problem. Moreover, though thin-walled members with holes are lately investigated by various researchers, cFEM approach is substantially different. In [23, 26, 27] the finite strip method is used, which is obviously different from finite element, and necessarily approximate in the presence of holes. In [28–30] shell finite element models are applied, but without constraints. In [31–34] the generalized beam theory is extended to members with holes, and though the mechanical background of these latter methods might be rather similar to the here-proposed cFEM, the practical realization of the generalized beam theory is quite different from that of a shell finite element analysis. It seems to be fair to state that shell finite element analysis is widely used by researchers and practitioners all over the world, therefore shell FEM is an obvious tool to be used in modal decomposition.

The cFEM has first been reported in [35–37]. The cFEM method is using shell finite elements, therefore, various engineering problems can be solved. Since cFEM uses a specific rectangular shell finite element, a rectangular mesh is necessary. This required regularity of the mesh means a practical limitation, but otherwise the method is general: first- and second-order static analysis as well as dynamic analyses can be performed, for arbitrary loading and boundary conditions. Holes can easily be handled, too, once they fit (exactly or approximately) into the rectangular mesh.

cFEM uses a novel shell finite element, specifically designed for the method. The new element keeps the transverse interpolation functions of finite strips, however, the longitudinal interpolation functions are changed from trigonometric functions (or function series) to classic polynomials. It is found, however, that the polynomial longitudinal interpolation functions must be specially selected in order to be able to perform modal decomposition similarly as in cFSM. This requires an unusual combination of otherwise well-known shape functions. The proposed interpolation functions and their derivation can be found in detail in [38].

In [38] it is also shown that the mechanical constraints on which the whole decomposition procedure is based can be satisfied exactly within the proposed shell finite element. The focus of the actual paper is on the modal decomposition for a member discretized into multiple shell elements. More specifically, in this paper the application of the constraints for a band of shell finite elements is presented, where ‘band’ is a segment of the member with multiple elements along the cross-section, but with one single finite element longitudinally. Though one band of finite elements rarely enough to solve a practical problem, the discussion of a band is important since (i) it is easier to present the constraining procedure and the resulted base systems if cFEM is applied for a band, and (ii) the base systems of a band can easily be applied for a real member consisted of multiple bands. This latter question is discussed in a paper companion to this paper [39], where numerical examples are also presented to prove the applicability and potentials of the constrained finite element method.

2. Converting a finite strip into a finite element

2.1. FSM essentials

The finite strip method (FSM) is a shell-model-based discretization method. The most essential feature of FSM is that there are two predefined directions, and the base functions (or: interpolation functions) are different in the two directions. In classical semi-analytical FSM, as in [40, 41], the structural member to be analysed is discretized only in one direction (say: transverse direction), while in the other direction (say: longitudinal direction) there is no discretization, i.e., in this direction there is only one element (i.e., strip) along the member, as shown in Fig. 1. (Note, Fig. 1 illustrates the nodal displacements for the simplest longitudinal shape function as given in Eqs. (1)–(3), with $m = 1$.)

In a strip it is typical to express the displacement functions as a product of transverse and longitudinal base functions. In the transverse directions polynomials are used, while in the longitudinal direction trigonometric functions can beneficially be used. Since there is no longitudinal discretization, the longitudinal interpolation function must well represent the behaviour, and especially, must satisfy the boundary conditions. If the end restraints are locally and globally pinned, the widely used FSM displacement functions are as follows (with using the notations of Fig. 1.

$$u(x, y) = \begin{bmatrix} (1-\frac{y}{b}) & (\frac{y}{b}) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} cos\frac{m\pi x}{a} \quad (1)$$

$$v(x, y) = \begin{bmatrix} (1-\frac{y}{b}) & (\frac{y}{b}) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} sin\frac{m\pi x}{a} \quad (2)$$