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Static and free vibration analyses of multilayered plates by a higher-order shear and normal deformation theory and isogeometric analysis

Loc V. Tran[a,b], Seung-Eock Kim[b,*]

[a] Department of Civil Engineering, Aalto University, P.O.Box 12100, FI-00076 Aalto, Finland
[b] Department of Civil and Environmental Engineering, Sejong University, 98 Gunja Dong Gwangjin Gu, Seoul 143-747, Republic of Korea

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ABSTRACT

This paper studies static and free vibration of multilayered plates based on isogeometric analysis (IGA) and higher-order shear and normal deformation theory. In which, the plate model with higher-order terms in the displacement fields can capture both shear deformation and thickness stretching effects. Consequently, it passes shear locking and achieves more accurate results in deflection, shear stress distributions, which are satisfied with traction free and interlaminate continuity conditions, and natural frequencies especially for sandwich plates. Utilizing non-uniform rational B-splines (NURBS) basis function fulfills C^1-continuity required by the plate model without additional variables.

1. Introduction

Due to some advantage characteristics, e.g. lightweight, high stiffness, long fatigue life, wear resistance and so on, multilayered structures are widely used in many engineering fields [1]. In which, the most common and well-known examples of layered structures are laminated composite plates as well as sandwich panels. Therefore, their behaviors related to deformable characteristic, stress distribution, dynamic response are always attracted to many scientists in the past few decades. For instances, Pagano [2] and Noor [3] proposed the analytical three-dimensional (3D) elasticity to predict the exact solution of static and dynamic problems for the composite and sandwich plates. Actually, the 3D elastic approach is a cumbersome object with high computational cost, mathematical complexity and inability to model the complex geometries under arbitrary boundary conditions. Alternatively, the 3D theory can be simplified to 2D plate theories by making suitable assumptions based on the equivalent single layer (ESL). In the context of ESL, the simplest classical laminate plate theory (CLPT) [4] merely provides acceptable results for the thin plates due to eliminating shear stresses, while the first order shear deformation theory (FSDT) [5,6] describes them incorrectly by a constant through the depth of each lamina. Hence, it requires an artificial shear correction factor (SCF) to rectify the unrealistic shear strain energy part. Afterwards, the higher order shear deformable theories (HSDTs) with higher-order terms in in-plane displacement approximations [7–9] have been proposed. In fact, without using SCF the HSDT models achieve better results and yield more accurate transverse shear stresses. In the aforementioned plate theories, with an assumption of constant transverse displacement approximation, thickness stretching effect is ignored. It means that $\sigma_z = 0$ and $\varepsilon_z = 0$. However, in some special cases, the transverse normal deformation is mandatory to accurately determine results for thick laminate plates. For instance, in the anisotropic multilayered plates having high Young's moduli ratio and low transverse shear moduli ratio, transverse normal flexibility is dominant with respect to in-plane deformability [10]. Eliminating the thickness stretching effect, thus, leads to misleading results, especially for thermally loaded plates [11,12]. An important role of the thickness stretch on analysis of multilayered plates and shells can be addressed in Carrera's works [13,14].

Quasi-3D theories have been therefore developed to remove the inconsistency of the aforementioned 2D plate models, in which the transverse displacement is also expanded with the higher-order variations with respect to the thickness coordinate. As a result, the transverse normal strain and stress can be captured. There are numerous quasi-3D theories introduced in literature. For example, Basset [15] suggested a Taylor series expansion of displacement field in the thickness coordinate. Kant and Swaminathan [16,17] gave analytical solutions for static and vibration analyses of laminated composite and sandwich plates using quasi-3D models with 12 and 9 unknowns. Matsunaga [18,19] proposed a Mth-order plate theory with $6M-1$ unknowns to investigate bending and dynamic problems of composite plate. Recently, the higher-order shear and normal deformation plate theories based on the Carrera Unified Formulation (CUF) were introduced to study the multilayered plates such as Carrera et al. [20] with 15 unknowns, Ganapathi and Makhecha [21] with 13 unknowns, Ferreira

* Corresponding author.
E-mail addresses: loc.tran@aalto.fi (L.V. Tran), sekim@sejong.ac.kr (S.-E. Kim).

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et al. [22] using 9 variables and so on. It is noteworthy that almost all the aforementioned quasi-3D models predict the plate behaviors with adequate accuracy. However, they are quite complex and computationally expensive due to generating extra unknown variables. Hence, several authors have been developed simplified quasi-3D plate theories to reduce the number of unknowns as well as governing equations. Zenkour [23] proposed a refined trigonometric quasi-3D theory for functionally graded plate (FGP) with four variables. Alternately, Thai et al. [24] extended this theory by using the invert trigonometric function instead. Thai and Kim [25] proposed a refined quasi-3D theory with only five unknowns for bending analysis of FGP, in which the transverse displacement is split into bending and shear parts. The refined theory produces acceptable results for isotropic and functionally graded plates but gains quite a considerable error for thick multilayered plates, as concluded in Kant's works [16,17] and Tran et al. [26]. Therefore, the generalized six-variable quasi-3D model proposal by Zenkour [27] is preferred to study the behavior of laminated composite plates. It is importantly noted that there is the first-order derivative of deflection in the approximated displacement fields. Thus, it is required at least C^1-continuity in approximation.

In finite element method, Hermite interpolation function with C^1-continuity was added for just specific approximation of transverse displacement. It produces extra unknowns related to the derivatives of the deflection ($w_{,x}$, $w_{,y}$, $w_{,xy}$) [7] which leads to increase in the computational cost. Thai et al. adopted meshfree method [28–30] in approximation to treat the stringent requirement without any additional variables. In his works, to impose essential boundary condition for clamped support, he assigned zero transverse displacement for two adjacent lines of nodes. This treatment creates a zone of zero deflection near the boundaries. By another way, NURBS based on IGA [31] is utilized to naturally satisfy the C^1-continuity. Because the basis function inherently owns the higher-order smoothness and continuity, up to C^{p-1} continuity by using pth degree. It is worth noting that rotation-free technique derived from IGA [32,33] enables us to impose the clamped condition with merely zero deflection on the boundaries edges. Owing to these salient features, IGA has been successfully implemented in many plate problems, for instance, linear and nonlinear analyses of composite plates [34–36], static, dynamic and buckling of FGPs [37–39]. It is stated that these plates in the above research are in simple geometries within a single-patch. Therefore, IGA easily handle C^1-continuity overall domain. However, in a complex shape described by several NURBS patches, the C^1-continuity is no longer maintained across the patch boundaries. This issue causes inaccuracy in bending (see an illustration in Ref. [40]), natural frequencies [24] and roughness of mode shapes (see Section 4.2.3) as well. Kiendl et. al. [32] proposed a bending strip method (BSM) for IGA to handle the C^1-continuity at the patch interfaces in the context of Kirchhoff-Love shell. This treatment is applied successfully for vibration analysis of thin plates [41,42].

The major contribution of this research is to develop an efficient computational approach based on IGA in association with a quasi-3D plate theory to study the static and free vibration of laminated composite and sandwich plates. The merits of the present approach are summarized as follows:

– By adding one variable in the transverse displacement, the present quasi-3D is an enhanced model of GSDT [44] with a possibility of capturing the thickness stretching effect. As a result, it achieves more accurate results in deflection, stresses and natural frequencies, especially for thick plates.
– The plate model is formulated in a general form based on a distributed function $f(z)$. By choosing type of $f(z)$, various higher-order plate theories, e.g., ESDT [8], SSDT [9], ITSDT [44], etc., can be retrieved.
– The proposed model is free of shear locking phenomenon without requiring any SCFs. Furthermore, the shear stresses, which are derived from the equilibrium equations [36], match well with the 3D solution and satisfy the traction free and interlaminate continuity conditions.
– NURBS based IGA is adopted to easily handle C^1-continuity of the quasi-3D model without additional unknowns. Hence, a number of unknowns are saved as compared to other C° quasi-3D theories. Moreover, BSM is embedded in IGA in order to model some complex shaped plates described in a multi-patch system.
– The reliability and accuracy of the present model are illustrated through comparisons with other available solutions in literature.

The paper is outlined as follows. Section 2 introduces the higher-order shear and normal deformation theory for the multilayered plates. Next, the isogeometric finite element implementation in the plate formulation is described. Section 4 provides the numerical results. Finally, this article is closed with some concluding remarks.

2. Higher-order shear and normal deformation theory for laminate composite plate

2.1. Displacement field

In the 2D plate theory, the displacement components are represented as a combination of the unknown variables at mid-plane of plate, $\Omega \subset R^2$ and the functions of thickness, z. According to this idea, a higher-order shear and normal deformation theory is suggested in a general form as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - zw_{0,x} + f(z)\phi_x(x,y) \\ v(x,y,z) &= v_0(x,y) - zw_{0,x} + f(z)\phi_y(x,y) \\ w(x,y,z) &= w_0(x,y) + g(z)\phi_z(x,y) \end{aligned} \tag{1}$$

where $z \in [-h/2, \quad h/2]$ is the thickness coordinate and $(\bullet)_{,i}$ denotes partial derivative along i-direction. Variables $\{u_0, \ v_0, \ w_0\}$ denote the axial displacements at the middle-plane, and $\{\phi_x, \ \phi_y\}$ are the rotations of normal vector of the middle-plane in the y-z, x-z planes, respectively. Meanwhile, an additional variable,ϕ_z, is introduced to account for the stretching effect. It makes sure of nonzero transverse normal strain ($\varepsilon_z \neq 0$). The coefficient of $\{\phi_x, \ \phi_y\}$ is given by an odd function $f(z)$, while that of ϕ_z is an even function,$g(z)$. For sake of brevity, the distributed function, $f(z)$, is chosen as a higher-order function as list in Table 1 to produce more accurate shear strain/stress distribution as compared with FSDT and $g(z) = f'(z)$ [27] in order to consider the thickness stretching effect.

2.2. Strain and stress components

Enforcing the assumptions of small strains and displacements, the strain-displacement relation is described as follows:

$$\begin{aligned} \boldsymbol{\varepsilon} &= [\varepsilon_x \varepsilon_y \gamma_{xy} \ \ \varepsilon_z]^T = \{\boldsymbol{\varepsilon}_1 0\} + z\{\boldsymbol{\varepsilon}_2 0\} + f(z)\{\boldsymbol{\varepsilon}_3 0\} + g'(z)\{\mathbf{0}\phi_z\} \\ \boldsymbol{\gamma} &= [\gamma_{xz} \ \ \gamma_{yz}]^T = f'(z)\boldsymbol{\varepsilon}_s + g(z)\boldsymbol{\kappa}_s \end{aligned} \tag{2}$$

where

Table 1
Distributed function and its derivative.

Model	$f(z)$	$g(z)$
TSDT [7]	$z - \frac{4}{3}z^3/h^2$	$1 - 4z^2/h^2$
ESDT [8]	$ze^{-2(z/h)^2}$	$(1 - \frac{4}{h^2}z^2)e^{-2(z/h)^2}$
SSDT [9]	$\frac{h}{\pi}\sin(\frac{\pi}{h}z)$	$\cos(\frac{\pi}{h}z)$
ITSDT [43]	$h\arctan(\frac{2}{h}z) - z$	$(1 - (\frac{2}{h}z)^2)/(1 + (\frac{2}{h}z)^2)$
[44]	$\frac{h}{\pi}\arctan(\sin(\frac{\pi}{h}z))$	$\cos(\frac{\pi}{h}z)/(1 + \sin^2(\frac{\pi}{h}z))$