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Influence of the deformation mode nature on the 1st order post-yielding strength of thin-walled beams

Nuno Silvestre[a,*], Miguel Abambres[b], Dinar Camotim[b]

[a] IDMEC, Department of Mechanical Engineering, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

[b] ICIST, CERIS, Department of Civil Engineering, Architecture and Georesources, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

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ABSTRACT

This paper presents a study on the influence of the deformation mode nature (global, local, distortional) on the load carrying capacity of beams beyond the yield load. Following recent investigations on the decomposition of elastic buckling modes into combinations of structurally meaningful deformation modes, this work applies the same concept to the 1st order failure modes (elastic-plastic collapse mechanisms). To achieve this goal, a GBT-based code that performs first-order elastic-plastic analyses of thin-walled members is employed. In order to study the influence of the mode nature on the post-yielding strength, five beams with different cross-sections, lengths, supports and loadings are analysed, and the results displayed by means of load-deflection curves, failure mode configurations and modal participation diagrams. On the basis of the limited study performed, it is concluded that larger contributions of local and distortional modes to the beam failure mode lead to a higher post-yielding strength reserve, which implies a higher beam load carrying capacity beyond the yield load. The opposite occurs for the contributions of global modes. Therefore, the member strength reserve obtained in geometrically non-linear analysis should not be credited only to the elastic post-buckling effects, but also to the plastic post-yielding effects.

1. Introduction

The behaviour of thin-walled steel members is genuinely non-linear, both physically and geometrically, and their strength and collapse mechanism are invariably governed by a combination of local and/or global plasticity and instability effects. Generally speaking, the kinematical definition of local and global deformation modes is nowadays clear: global modes are characterised by rigid-body motions of the member cross-sections (*e.g.*, transverse translations, associated with bending or twist rotations, associated with torsion) while local modes are characterised by cross-section in-plane deformations (wall bending). Regarding the local modes, it is still possible to distinguish between local-plate (wall transverse bending with no corner in-plane motions) and distortional (wall transverse bending combined with rigid-body motions of cross-section parts involving corner in-plane motions) modes. Usually, the local-plate modes are merely termed "local modes".

The sources of nonlinearity can be instability and/or plasticity. If linear elasticity is considered (plasticity effects excluded), the characterisation of the geometrically non-linear behaviour of a thin-walled member usually requires not only the determination of the critical buckling load (F_{cr}) but also the qualitative (indirect) assessment of its post-buckling stiffness (K_{PB}). It is well known that the post-buckling stiffness (K_{PB}) of a thin-walled member is highly dependent on the nature of its critical buckling mode: (i) low K_{PB} if the mode is global (G), (ii) high K_{PB} if the mode is local (L), (iii) moderate K_{PB} if the mode is distortional (D) – see Fig. 1, concerning a lipped channel column under a compressive force F. In case of a mixed buckling mode (L+D+G), it is possible to anticipate qualitatively the level of post-buckling stiffness K_{PB} depending on the modal participations [1]. The classification of buckling modes has been widely investigated over the last two decades using GBT, cFSM and cFEM [2–8].

Much research has been done on the classification of buckling modes from the elastic instability side. However, nothing has been done on the plasticity side. If only plasticity is considered (instability effects excluded), the characterisation of the physically nonlinear behaviour of a thin-walled member usually requires the determination of not only the yield load (F_y) but also the plastic load (F_p) – note that F_y stands for the load corresponding to the first yield (limit of elastic behaviour) while F_p denotes the maximum load, corresponding to the collapse

* Corresponding author.
E-mail address: nsilvestre@ist.utl.pt (N. Silvestre).

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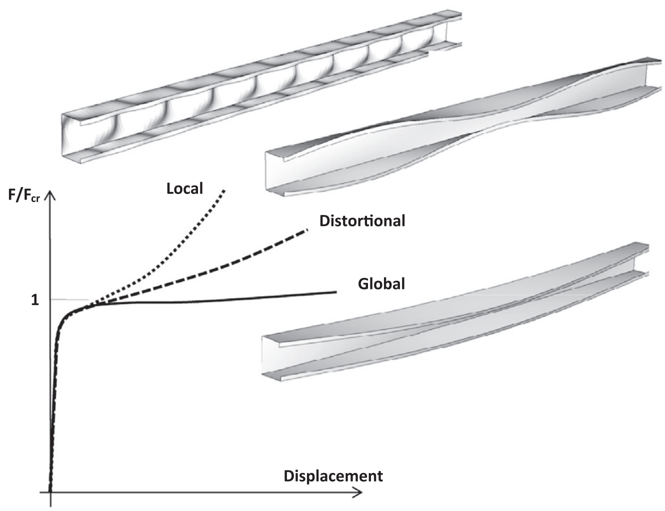

Fig. 1. Post-buckling stiffness of different modes (column behaviour).

(plastic) mechanism ($F_p \geq F_y$ – see Fig. 2(a)). Like in the case of instability (post-buckling stiffness K_{PB}), a good measure of the capacity of a member to develop plastic deformations is the so-called "post-yielding strength", which should be viewed as the ratio between the maximum plastic load (horizontal plateau[1]) and the yield load (see Fig. 2(a)),

$$S_{PY} = F_p/F_y \tag{1}$$

In the case of pure plastic behaviour, there is no information on the influence of the deformation mode nature on the level and magnitude of the post-yielding strength S_{PY}. Should the local (L) and distortional (D) modes exhibit a higher post-yielding strength than the global modes (see Fig. 2(b))? The purpose of this paper is to answer this question.

In order to achieve this goal, a first-order elastic-plastic GBT formulation developed and numerically implemented by the authors [9,10] is adopted to perform the analyses. The GBT-based concept of modal decomposition of thin-walled member deformed configurations in the elastic-plastic range, including collapse mechanisms, is adopted. The most relevant modal results addressed consist of load-deflection curves, determined by means of GBT analyses that include only pre-selected deformation mode sets, and modal participation diagrams. Finally, some concluding remarks are drawn regarding the effect of the mode nature on the post-yielding strength of the beams (through parameter S_{PY}).

2. Modal participation in GBT

The GBT formulation and its MATLAB implementation to perform first-order elastic-plastic analyses are not presented here but can be found elsewhere − the interested reader may find detailed information about this formulation in references [9,10]. This paper will focus on the definition of mode participation factor rather than on the GBT formulation. Consider the local coordinate system (x, s, z), where x, s and z are, respectively, the longitudinal ($0 \leq x \leq L$), mid-line transverse ($0 \leq s \leq b$) and through-thickness ($-t/2 \leq z \leq t/2$) coordinates – L is the member length, b is the wall width and t is the wall thickness. The corresponding local displacements are u (along x – warping), v (along s – transverse) and w (along z – flexural). The GBT analysis of a structural member consists of a cross-section analysis and a member analysis. The cross-section analysis considers four deformation mode families: conventional (global, local and distortional) modes, warping shear modes, transverse extension modes and cell shear flow modes (the last ones only in cross-sections with closed cells). These deformation mode families are obtained by solving sequences of eigenvalue problems. Each deformation mode is associated with a unique displacement profile, involving in-plane ($v_k(s)$ and $w_k(s)$) and out-of-plane ($u_k(s)$ − warping) displacements, all functions of the mid-line coordinate s. For illustration purpose, a set of deformation modes of an I-section with hollow flanges (usually designated as "dog-bone") obtained from GBT cross-section analysis is shown in Fig. 3.

The member analysis comprises the determination of the modal amplitude functions $\zeta_k(x)$ that provide the variation of each deformation mode amplitude along the member axis (coordinate x). In a GBT analysis, the displacement field at the member mid-surface is expressed as

$$\begin{aligned} u(s,x) &= u_k(s)\zeta_{k,x}(x) \\ v(s,x) &= v_k(s)\zeta_k(x) \\ w(s,x) &= w_k(s)\zeta_k(x) \end{aligned} \tag{2}$$

where $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the (normalised[2]) deformation mode displacement profiles, $\zeta_k(x)$ are the corresponding modal amplitude functions and the summation convention applies to subscript k. Thus, the displacement field associated with any member deformed configuration (*e.g.*, a buckling mode or a collapse mechanism) is expressed as a linear combination of products involving modal displacement profiles and their longitudinal amplitude functions.

In GBT, Eq. (2) can be used to separate the contributions of all deformation modes to a given member deformed configuration – reached in the elastic or elastic-plastic regimes. The contribution (participation) of a given mode i to a cross-section deformed configuration, designated as c_i, is usually quantified by means of the ratio between the corresponding ζ_i value and the sum of all such values, *i.e.*, one has

$$c_i = \frac{|\zeta_i|}{\sum_{k=1}^{N_{GBT}} |\zeta_k|}, \tag{3}$$

where N_{GBT} is the total number of deformation modes included in the GBT analysis. Obviously, since the values of $\zeta_1, \zeta_2, \ldots, \zeta_{NGBT}$, vary differently along the member axis (x coordinate), the value of the ratio defined by Eq. (3) also varies along the member length. Therefore, it seems logical to quantify this contribution by means of a participation factor (p_i) defined as[3]

$$p_i = \frac{\int_0^L |\zeta_i|}{\sum_{k=1}^{N_{GBT}} \left(\int_0^L |\zeta_k|\right)}. \tag{4}$$

These modal participation definitions were often used in the context of GBT elastic buckling and post-buckling analyses [1–4]. Unlike elastic buckling modes (global, local, distortional), which are known to involve the deformation of the whole member, the elastic-plastic collapse mechanisms are often associated with highly localised deformation patterns. Thus, the usual GBT participation factor definition provided in Eq. (4) is not adequate to characterise elastic-plastic failure modes and, therefore, will not be adopted in this work. Instead, the following methodology is adopted for an arbitrary member equilibrium configuration:

- Calculate the overall displacement field (d (s, x)) in the whole member mid-surface, by means of (k satisfies the summation convention)

[1] Note that 2nd order (or buckling) effects are excluded and there is neither local maximum (peak) nor descending path of load-displacement curve.

[2] The axial extension and warping shear deformation modes are normalised so that they exhibit a unit maximum axial displacement. All other deformation modes are normalised to exhibit a unit maximum in-plane displacement.

[3] Obviously, the axial extension and warping shear mode amplitude functions in Eqs. (3) and (4) should read $\zeta_{k,x}$ instead of ζ_k (recall Eq. (2)).