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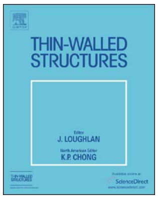

Full length article

Length effects on interactive buckling in thin-walled rectangular hollow section struts

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ABSTRACT

A variational model formulated using analytical techniques describing the nonlinear coupling between local and global buckling modes within an elastic thin-walled rectangular hollow section strut is presented. A system of nonlinear differential and integral equations subject to boundary conditions is derived and solved using numerical continuation techniques. The nonlinear behaviour of four representative lengths is investigated, which are characterized by the post-buckling equilibrium paths. The numerical results from the variational model are validated using a nonlinear finite element model and largely show excellent comparisons, particularly for the practically important ultimate load and the initial post-buckling behaviour. Boundaries for the four distinct length-dependent zones are identified and the most unstable zone is demonstrated to have a considerably narrower length range than previously determined for practical corner boundary conditions within the cross-section.

1. Introduction

The development of structural material and manufacturing technology alongside fundamentally better understanding of nonlinear mechanics have enabled structural forms to become increasingly slender. Buckling instability is practically always the governing failure mode of such structures [1–4]. Nonlinear elastic stability theory [5,6] has validated that slender columns and plates designed with linear theory are safe owing to their naturally stable post-buckling characteristics. According to linear theory, an equal stress triggering both local and global buckling would often seem to represent the most economical scheme. However, such approaches have been found to be inappropriate for thin-walled slender structures particularly when they become vulnerable to a variety of different nonlinear buckling phenomena [7–14]. Even though these modes may be stable when triggered in isolation, the interaction between individual modes can lead to unstable post-buckling behaviour and severe imperfection sensitivity [15–28].

Early work on the interactive buckling of columns was conducted by van der Neut [8]. Using the theory originally devised by Koiter [5], van der Neut developed a relatively simplified model comprising two load-carrying flanges, which were simply-supported along their long edges, and a pair of rigid webs that only served to maintain the structural integrity but provided no longitudinal stiffness, as shown in Fig. 1(a). Using this simplified model, the relationship between the properties of the initial post-buckling behaviour and the ratio of the global buckling load $P_{\mathrm{o}}^{\mathrm{C}}$ and the local buckling load $P_{\mathrm{l}}^{\mathrm{C}}$, referred to currently as the 'van der Neut curve', was obtained, as shown in Fig. 1(b).

Using largely the same approach, Koiter and Pignataro [29] investigated local–global mode interaction and developed an equivalent to the van der Neut curve for stiffened panels with one or multiple stringers. Compared with the idealized column, it was found that the unstable post-buckling range, labelled as zone 2 in Fig. 1(b), is considerably smaller in stiffened panels. Owing to the limitations of the asymptotic method used, only very restricted information about the nonlinear behaviour was presented apart from the initial post-buckling response.

Byskov [30] applied the asymptotic expansion method developed previously in [20] to van der Neut's column. However, owing to the limitation of the method [31,32], which is only valid when the post-buckling behaviour of the local mode is neutral or unstable, the errors were relatively large and the results were only meaningful in close proximity of the boundary between zones 1 and 2.

Sridharan and his collaborators [33–36] used the finite strip method [37,38] in conjunction with Koiter's theory to analyse mode interaction in thin-walled structural members with different cross-sections. As for the global mode, both the purely flexural and the flexural–torsional buckling modes were considered. The numerical results showed good comparisons with existing experimental results and substantial erosion in load-carrying capacity due to imperfections was identified. Using a similar approach, Møllmann and Goltermann [39,40] investigated the

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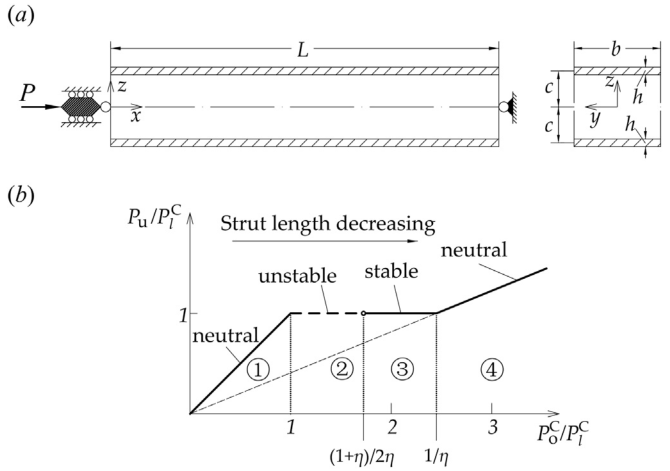

Fig. 1. (a) The van der Neut model of an idealized thin-walled strut [8]. The strut comprises two load-carrying flanges with width b, thickness h, length L with a pair of rigid webs of depth $2c - h$ with no longitudinal stiffness; P is the concentric axial load applied to the entire strut. (b) The van der Neut curve for the geometrically perfect case: η is the stiffness reduction factor due to the local buckling of the flanges; P_u, P_o^C and P_l^C are the ultimate, the global buckling and the local buckling loads respectively.

interactive buckling behaviour of an I-section beam under pure bending and a box-section column under pure compression. Substantial reductions of up to 50% in the load-carrying capacity were observed in both examples due to the mode interaction.

The interactive buckling of square hollow section columns using the commercial finite element (FE) package ABAQUS was investigated in [41]. Three example columns with typical slendernesses were analysed, where (i) global buckling and (ii) local buckling were clearly critical, and (iii) where global and local buckling were triggered simultaneously. Unstable post-buckling paths resulting from the mode interaction were observed in all the example columns. The localization of the post-buckling mode due to the interaction was also highlighted. Since that work principally focused on the effects of initial imperfections, residual stresses and material plasticity on the ultimate load and post-buckling behaviour, no further details about the evolution of the modal interaction, equilibrium paths, slenderness effects nor the underlying mechanism were presented.

More recently, Wadee and his collaborators have developed a series of variational models using analytical techniques to investigate mode interaction in I-section struts [13,42] and stiffened plates [14]. These variational models exhibit very good comparisons with FE models using shell elements and existing experimental results [11]. Using the numerical continuation method, the evolution of the modal interaction in the post-buckling range has been captured well by these models. In particular, for some geometric ranges, snap-backs in the response, showing sequential destabilization and restabilization and a progressive spreading of the local buckling mode from the mid-span, known as cellular buckling [43], have been predicted, which have also been observed in physical experiments on thin-walled struts [11] and beams [12]. Using the validated variational models, the effects of the global and local slenderness on the mode interaction behaviour have been investigated. Equivalent van der Neut-type curves for I-section struts [27,44] and stiffened plates [26], with more detailed information about the post-buckling behaviour has been presented, thus identifying the most unstable parametric spaces of the geometries.

Mode interaction has also been observed in physical experiments on hollow section columns [45–50]. However, these particular investigations have principally focused on the ultimate load capacity and the behaviour in the inelastic range. Therefore, limited information on the underlying mechanisms of mode interaction has been presented. However, in the physical tests on I-section struts under pure compression [51,11] and I-section beams under uniform bending [12], more details about the evolution of the interactive buckling, in terms of the amplitude modulation and the change in the wavelength of the local buckling profile with the increase of the applied loading deformation, were highlighted.

In the authors' recent work [52], a variational model describing mode interaction in long rectangular hollow section struts was developed and showed excellent comparisons with FE models developed within the commercial package ABAQUS [53]. The work identified that when the local buckling load is close to the global buckling load, such a system can exhibit a violent destabilization once the peak load is reached. The present work develops the model to include the scenarios where local buckling is critical to investigate how shorter strut lengths affect the nonlinear modal coupling.

The interactive buckling responses of four representative struts that are currently defined qualitatively in sequence as the length is progressively reduced as 'long', 'transitional', 'intermediate' and 'short' length struts, thus corresponding to the four different zones shown in Fig. 1(b) respectively, are studied. The characteristic interactive buckling paths are determined and a progressive change in the local buckling wavelength is found. The numerical results from the variational model largely show excellent comparisons with numerical results obtained using a nonlinear FE model developed within ABAQUS in terms of the ultimate load and the initial post-buckling behaviour. A parametric study on the strut length identifies the boundaries of the four distinct length-dependent domains and places the current results within the context of the classical work by van der Neut. The current work facilitates a better understanding of the behaviour of thin-walled rectangular hollow section struts that exhibit local–global mode interaction with different slendernesses and in future will allow for the establishment of guidance for structural engineers designing such elements that are vulnerable to such phenomena.

2. Development of the variational model

A thin-walled rectangular hollow section strut of length L with simply-supported boundary conditions being loaded by a concentric force P is considered, as shown in Fig. 2. The web depth and thickness are d and t_w respectively; the flange width and thickness are b and t_f

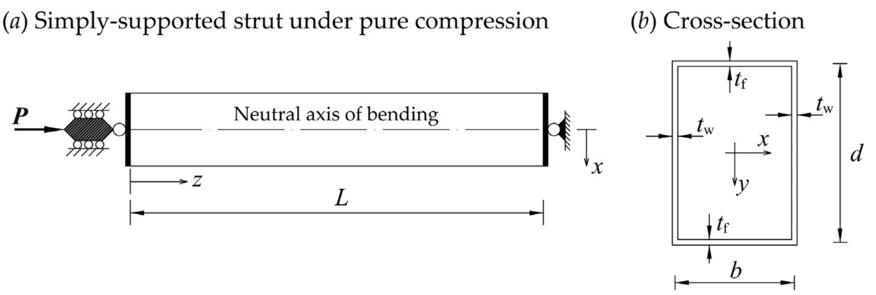
(a) Simply-supported strut under pure compression (b) Cross-section

Fig. 2. (a) Plan view of the rectangular hollow section strut of length L under the concentric axial load P. The lateral and longitudinal coordinates are x and z respectively. (b) Cross-section geometry of the strut; the vertical axis coordinate being y.