Thin-Walled Structures 127 (2018) 31–39

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

A theoretical model of the inversion tube over a conical die

YuZhe Liu[a], XinMing Qiu[a,*], T.X. Yu[b,c]

[a] Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
[b] Department of Mechanical and Aerospace Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
[c] Y.K. Pao Chair Professor, Ningbo University, Ningbo, China

ARTICLE INFO

Keywords:
Theoretical model
Inversion
Energy absorption
Plastic dissipation
Friction

ABSTRACT

As an important impact energy absorber and a way to produce double-wall tubes, the inversion of the metal tube over a conical die is investigated in this study. Adopting the deformation profile inspired by FEM simulations, a theoretical model is proposed assuming the energy is dissipated by bending and compression in the meridional direction, and expansion in the circumferential direction. FEM simulations with a large range of geometrical parameters are used to validate the theoretical model, and results show that both the compressional force and the final circular radius of tube predicted by the theoretical model are quite accurate. Then, the energy dissipations along the axis of the tube by various mechanisms are analyzed in detail, and it is identified that the tube deformation in the thickness direction is the major reason of the current model's deviation. Besides, the influence of the die radius, the friction between die and tube, as well as the dynamic effect are discussed.

1. Introduction

Thin-wall metal tubes compressed over a rigid die are widely used in the impact protection and the forming industry [1–4]. Due to different geometry of the die, the deformation of tubes could be expansion [5–12], reduction [13,14], external or internal inversion [2,3,15–23].

The expansion and the reduction of the metal tubes are used to produce the tubes with the specific radii [13]. Besides, the expansion is also desirable for the impact energy absorber because of these advantages: steady compressional force, high Specific Energy Absorption (SEA), insensitivity to the load direction and less constraint on the ductility of the tube material [6]. In most of the studies, the conical die was adopted in the expansion, while the die with an arc profile was investigated by Lu et al. [24]. The process of the expansion and inversion were studied by experiments and simulations [7,9], and based on the equilibrium equations, several theoretical models were proposed considering the friction, variation of the tube thickness and the distribution of the contact pressure [5,8,10]. Based on the deformation theory, Liu et al. [11,12] proposed a simple theoretical model which provided the accurate predictions of the compressional force and the expanded radius within a wide range of the geometrical parameters.

The inversion of metal tubes is also used as the impact energy absorber and to produce the double-wall tube. Compared with expansion, the inversion of tube has higher stroke efficiency (deformed length/initial length) because the die in expansion is almost as long as the stroke [25]. Most studies of the inversion tube concerned on the die with arc profile, e.g., Sekhon et al. [15] measured the steady compressional forces and the deformation profiles of the tubes, and it was found that the compressional force decreased with the increasing arc radius of the die. Reddy [19] assumed the deformation profile of the tube was a circle and calculated its radius by minimizing the energy dissipations. Based on the equilibrium equations in the tangential and the radial directions, Miscow et al. [20] proposed a theoretical model to give the distribution of the contact pressure and the compressional force versus loading displacement. However, the deviation of this model from the experimental results was large, so it can only be used to estimate the parameters of the inversion. Then Niknejad et al. [21] improved the Miscow's model by considering the curling at the free end of the tube, and divided the deformation process into three stages according to the positions of the tube end. Al-Hassani et al. [16] performed several experiments about both the external and internal tube inversions. Considering the energy dissipation, they proposed a theoretical model with the frictionless contact, and the compressional force given by the model matched well with the experiments. Leu [17] investigated the critical conditions between flaring and curling by comparing the increments of total deformation energy, and found that the strain-hardening effect and half-apex angle of the die were the major factors and the friction coefficient was the minor factor. Assuming that the radius of the curvature at the free region was different from the contact region, Yu et al. [26] proposed a theoretical model with the simple forms and improved the accuracy compared with the previous models. Recently, Omid et al. [23] investigated the response of variable

* Corresponding author.
E-mail address: qxm@tsinghua.edu.cn (X. Qiu).

https://doi.org/10.1016/j.tws.2018.01.035
Received 23 June 2017; Received in revised form 7 December 2017; Accepted 24 January 2018
0263-8231/

thickness distribution inversion tube under the oblique loading by numerical simulation and experiments, and gave the influences of the die filet radius, frictional coefficient and the distribution of the thickness on the energy absorption. Besides the inversion with a circular die, the free inversion is also used as energy absorbers [27–34]. Guist et al. [27] proposed the original theoretical model of the free inversion with the constant curvature and thickness assumptions. Then, Kinkead [28] adopted the energy dissipated by the bending in the circumferential direction and Qiu et al. [29–31] considered the change of the thickness and the curvature in the model to improve the prediction of the steady compressional force. Recently, the fiber-reinforced composite is used in the design of the energy absorbers for its high strength-weight ratio [32,33], and the free inversion of the composite reinforced tube is investigated theoretically and numerically by Guo [34].

The inversion with the conical die can also be applied in the impact protection and the forming industry. Due to its simple shape, the conical die can be easily produced and then used with different sizes of tubes. However, very limited study focused on the inversion over a conical die. Sun et al. [18] investigated the critical conical angle of the die between the flaring and the curling, and it was found that the strain-hardening effect of the tube material was the most important factor. Masmoudi et al. [35] performed the experiment and simulation study about the inversion with the conical die and analyzed the deformation modes in detail. In order to guide the symmetric steady inversion, a small curvilinear transition part was made between the cylindrical and conical parts of the die.

To authors' best knowledge, no theoretical model predicting the steady compressional force and the final circular tube radius has been reported. Therefore, the steady inversion of the metal tube over a conical will be analyzed in this study.

In Section 2, an axisymmetric deformation profile is assumed, and the energy dissipation by the expansion in the circumferential direction and the bending in the meridional direction are adopted in the basic model. Then, based on the equilibrium equations, the compression in the meridional direction is taken into account to construct a refined model. And the influence of the friction is adopted considering the balance of the die. In Section 3, the experiment result reported in previous study [35] is compared with current model, and FEM simulations are performed to validate the accuracy and applicability of current models. Then the energy dissipations of various deformation mechanisms along the axis are analyzed in Section 4. As results, the energy dissipation by the bending in the circumferential direction is confirmed to be negligible, and the deviation of the energy dissipation by the compression in the meridional direction is found to be mainly caused by the constant thickness assumption. Finally, the deformation of the tube before the steady stage, the influence of the die radius and the dynamic effect are discussed in Section 5.

2. Theoretical model

In this section, the steady inversion process of a metal tube over a rigid conical die is analyzed. As shown in Fig. 1**a**, the conical die is compressed onto the tube with a constant velocity V_0, and the tube is inverted after the contact point with the die. Points A, B, C represent the start point of deformation, the contact point with the die, and the point of deformation termination, respectively. The corresponding circular radii of the tube at points A, B, C are r_0, r_1, r_2, respectively. The conical angle of the die is α, and the die radius r_{die} is assumed to be large enough, i.e., $r_{\text{die}} > r_2$. The tube thickness t_0 is assumed to be unchanged during the deformation process. Firstly, the contact between the tube and the die is assumed to be frictionless, and the effect of friction will be discussed later. A material coordinate system S is defined along the tube axis, while $S = 0, S_1, S_1 + S_2$ are associated with points A, B, C, respectively.

For simplicity, the tube material is assumed to be rigid, perfectly plastic material (R-PP), which applies to material that has no obvious strain hardening effect such as aluminum, brass and steel [36]. The yield strength of the tube material is denoted as Y. Except for friction, the energy is mainly dissipated by four plastic deformation mechanisms: bending and compression in the meridional direction (denoted by subscript l); expansion and bending in the circumferential direction (denoted by subscript ϕ), respectively. According to the energy conservation of the rate form,

$$F \bullet V_0 = \dot{E} = \dot{E}_l^{\text{B}} + \dot{E}_l^{\text{C}} + \dot{E}_\phi^{\text{B}} + \dot{E}_\phi^{\text{E}} \tag{1}$$

where F is the steady compressional force; V_0 is the constant loading velocity; $\dot{E}$ is the total energy dissipation rate; $\dot{E}_l^{\text{C}}$ and $\dot{E}_l^{\text{B}}$ are the energy dissipation rates of bending and compression in the meridional direction, respectively; $\dot{E}_\phi^{\text{E}}$ and $\dot{E}_\phi^{\text{B}}$ are the energy dissipation rates of expansion and bending in the circumferential direction, respectively.

During the steady compression process, over a short time duration $\delta\tau$, the energy dissipation rate of each deformation mechanism is,

$$\begin{cases} \dot{E}_l^{\text{B}} \bullet \delta\tau = \int_0^{S_1+S_2} 2\pi r \bullet M_l \bullet \delta\kappa_l \bullet \mathrm{d}S \\ \dot{E}_l^{\text{C}} \bullet \delta\tau = \int_0^{S_1+S_2} 2\pi r \bullet N_l \bullet \delta\epsilon_l \bullet \mathrm{d}S \\ \dot{E}_\phi^{\text{B}} \bullet \delta\tau = \int_0^{S_1+S_2} 2\pi r \bullet M_\phi \bullet \delta\kappa_\phi \bullet \mathrm{d}S \\ \dot{E}_\phi^{\text{E}} \bullet \delta\tau = \int_0^{S_1+S_2} 2\pi r \bullet N_\phi \bullet \delta\epsilon_\phi \bullet \mathrm{d}S \end{cases} \tag{2}$$

where M is the bending moment, N is the membrane force, κ is the curvature, ϵ is the membrane strain. Similar to Eq. (1), the contribution component of each deformation mechanism to the steady compressional force is defined as,

$$F_l^{\text{B}} = \frac{\dot{E}_l^{\text{B}}}{V_0}; F_l^{\text{C}} = \frac{\dot{E}_l^{\text{C}}}{V_0}; F_\phi^{\text{B}} = \frac{\dot{E}_\phi^{\text{B}}}{V_0}; F_\phi^{\text{E}} = \frac{\dot{E}_\phi^{\text{E}}}{V_0} \tag{3}$$

where F_l^{B} and F_l^{C} are force contributions by bending and compression in the meridional direction, respectively; and F_ϕ^{B} and F_ϕ^{E} are force contributions by bending and expansion in the circumferential direction, respectively.

From FEM simulations that will be given in the following section, it is confirmed that $\dot{E}_\phi^{\text{B}}$ is negligibly small. Therefore, in our first step only the components of $\dot{E}_l^{\text{B}}$ and $\dot{E}_\phi^{\text{E}}$ are adopted in the basic theoretical model. Furthermore, in order to improve the predication accuracy in the case of large conical angle ($\alpha \geq 75°$), $\dot{E}_l^{\text{C}}$ will be taken into account to construct a refined model.

2.1. Basic model

It is seen from the deformation profile given by FEM simulation, as shown in Fig. 1**b**, that the curvature along the meridional direction κ_l are varying. In segment AB, κ_l is almost unchanged that is assumed to be a constant κ_0. As shown in Fig. 1**a**, the center of arc AB is point O, and the curvature radius is $b = 1/\kappa_0$. While in segment BC, κ_l gradually decreases from κ_0 (at point B) to 0 (at point C), so a linear distribution of κ_l along S is adopted,

$$\kappa_l(S) = \begin{cases} \kappa_0 \quad 0 \leq S \leq S_1 \\ \kappa_0(S_1 + S_2 - S)/S_2 S_1 \leq S \leq S_1 + S_2 \end{cases} \tag{4}$$

The curved angles at points B and C are α and π, respectively; since the tube profile is tangential to the die at point B, and fully inverted at point C. Consequently, the lengths of arcs AB and BC are $S_1 = \alpha/\kappa_0$ and $S_2 = 2(\pi - \alpha)/\kappa_0$. In Fig. 2, the assumed curvature distribution $\kappa_l(S)$ is compared with the FEM simulation results. Clearly, the current model catches most prominent features of the deformed profile. Besides, the assumed deformation profile only depends on κ_0, whose value will be decided in the following.

Considering a small increment of tube segment $\mathrm{d}S$, the corresponding increment of the curved angle θ and the circular radius r are,

$$\mathrm{d}\theta = \kappa_l \bullet \mathrm{d}S \tag{5}$$