Thin-Walled Structures 129 (2018) 381–390

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Free vibration of thin rectangular steel plates with geometrically-nonlinear load-displacement behavior

Ali Rezaiefar, Khaled Galal*

Department of Building, Civil and Environmental Engineering, Concordia University, Montréal, Québec, Canada H3G 1M8

ARTICLE INFO

Keywords:
Rectangular plate
Finite element
Modal analysis
Vibration
Large deformation
Significant frequency
Mass participation

ABSTRACT

This paper provides means for obtaining the first three *significant* vibration modes for rectangular plates based on mass participation ratios. A non-dimensional frequency parameter is presented which results into the vibration frequency of rectangular plates at each of these three significant modes. Various aspect ratios and four combinations of boundary conditions at the plate edges are studied. A correlation between the nonlinear load-deformation behavior of the plate and its vibrational behavior is also presented accordingly. It is demonstrated that the vibration frequency of the studied rectangular plates increases significantly upon increasing the applied lateral pressure if the large deformation effects are considered in the analysis. The easy-to-follow method of frequency calculation presented in this paper is useful for assessing the dynamic characteristics of rectangular plates with or without lateral pressure that are subject to vibration.

1. Introduction

Stiffened plates with rectangular panels are one of the most common forms of thin walled structures in industrial, naval and aerospace structures. The walls of industrial ducts, ship hauls, rectangular bins and aircraft wings are examples of such plates. The vibration of rectangular plates has gained the attention of many researchers in the past decades. Structural vibration analysis involves studying the vibration properties of any structure that is subjected to any form of vibrating force. Resonance conditions where the forcing frequency and the natural frequency of the structure are the same (or very close) are to be avoided. Although there are no code regulations about the resonance checking, structural engineers tend to set the limits of $\pm$ 20% as the resonance domain, i.e. if the forcing frequency is within the range of 0.8–1.2 times the natural frequency, the structure is considered as *"prone to resonance"*. However, there are still questions to be answered e.g.: "How many modes should be considered?" and "Which modes are important?" that are involved with the vibration analysis of structures. Answering these questions for the cases of single-degree-of-freedom (SDOF) and multi-degrees-of-freedom (MDOF) structures involves the measurement of the participation of the structural mass in the desired DOFs. For the case of SDOF systems, there is only one mode of vibration and the mass participation for that mode is merely 100%. The vibration frequency at this mode is often called the *"fundamental frequency"*. Similarly, The fundamental frequency in MDOF systems corresponds to the mode with the highest mass participation which is usually achieved at the first degree of freedom.

While the main aspects of the structural design of rectangular plates are the load-deformation and load-stress behavior, vibration-induced limit states such as fatigue and loosening of connections might cause unforeseen deficiencies during the service life of the structure if it is prone to resonance with the applied dynamic loads.

Several studies in terms of closed-form solutions and numerical methodologies to calculate the frequencies of vibration of rectangular plates are available in the literature including Gorman [13], Soedel [35], Amabili [2] and Chakraverty [6]. Effects of several parameters on the natural frequency of rectangular plates are the topic of further research including the studies by Liew et al. [19] wherein the effects of in-plane isotropic pressure on the vibration response of thick rectangular plates were discussed. More recently, Xiang et al. [43] presented one of the first known exact solutions for the vibration of rectangular multi-span plates with two opposite edges simply supported using the Levy type solution method and the state-space technique. Phillips and Jubb [27] presented a series of tests to verify the natural frequency of vibration of an approximately clamped rectangular plate due to increasing lateral distortion against the existing theory initially presented by Reissner [31]. The initial distortion of the plate in their experimental tests was applied by deflecting the plate using a rigid block at the center of the plate and a hydraulic jack prior to the vibration test. Manzanares-Martínez et al. [22] presented the lower normal modes of vibration of rectangular plates through experimental and theoretical analysis. Their experimental tests involved employing electromagnetic-acoustic

* Corresponding author.
E-mail address: khaled.galal@concordia.ca (K. Galal).

https://doi.org/10.1016/j.tws.2018.02.032
Received 29 March 2017; Received in revised form 25 January 2018; Accepted 28 February 2018
0263-8231/ © 2018 Elsevier Ltd. All rights reserved.

Nomenclature

B	breadth of rectangular plate
c	viscous damping constant
D	flexural rigidity of plate material
E	modulus of elasticity of plate material
f	external excitation as a function of time
h	thickness of plate
L	length of rectangular plate
N_x	membrane force in x direction
N_y	membrane force in y direction
N_{xy}	shear force in xy plane
q	lateral pressure applied to the plate surface
q*	non-dimensional lateral pressure
S	plate aspect ratio
Δ	maximum deflection at the center of rectangular plate
ω*	non-dimensional frequency parameter
ω	vibration frequency of plate
ν	Poisson's ratio of plate material
ρ	mass density per unit volume of material
w(x,y,t)	dynamic displacement of a given point on plate surface (x,y) at specific time (t)
$\dot{w}$	velocity of any given point on plate surface (x,y) at specific time
$\ddot{w}$	acceleration of any given point on plate surface (x,y) at specific time

transducers on a thin rectangular plate in order to measure the frequencies of vibration mainly in the acoustic modes with high frequencies.

Cho et al. [8] studied the vibration of rectangular plates for the case of bottom plate of fluid containers where the plate is in direct contact with fluid. The fluid-structure interaction was the focus of that study using the Lagrange's equation of motion to relate the potential and kinetic energies of the plate structure and surrounding fluid kinetic energy. The study of the effects of imperfections on the vibration of rectangular plates was further extended by Zeng et al. [45] where a loaded side-cracked plate was studied using the Moving-Least-Square (MLS-Ritz) method. Huang and Lin [14] also studied the vibration of rectangular plates with a straight-through crack using Fourier cosine series with domain decomposition. Lately, Wang et al. [41] studied the effects of rectangular openings on the vibration characteristics of rectangular plates in order to establish a methodology for noise reduction and vibration control in thin-walled structures with openings using Fourier series.

It has to be mentioned that the concept of *nonlinear vibration of plates* that includes the response of plates to periodic loading with large amplitudes is out of the scope of this paper. This paper mainly focuses on the free vibration and fundamental frequencies of rectangular steel plates with various aspect ratios and the effects of large deformations caused by static lateral pressure on those frequencies.

2. Theoretical background, FE modeling and verification

The earliest attempts on the vibration analysis of rectangular plates in presence of in-plane (membrane and shear) forces were made by Dawe [9] and followed by alternative solutions including numerical methods by Mei and Yang [23], Dickinson [10], and Bassily and Dickinson [4]. Chan and Foo [7] were amongst the first to apply the *Finite Strip method* to solve the case of rectangular plate vibration with membrane forces.

The general equation of motion for a rectangular plate under lateral and in-plane loading is presented in Eq. (1).

$$D\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]^2+ch\dot{w}+\rho h\ddot{w}=f+q+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y} \quad (1)$$

In which D is the flexural rigidity of the plate calculated as $D = Eh^3 / 12(1-\upsilon^2)$, c is the viscous damping, ρ is the mass density per unit volume of plate material, h is the plate thickness and $w(x, y, t)$ is the dynamic displacement at any given point of plate surface at specific time *(t)*. The first and second derivatives of w with respect to t (i.e. $\dot{w}$ and $\ddot{w}$) represent the velocity and acceleration at a specific time. On the right side of Eq. (1), f is the external excitation as a function of time, q is the lateral pressure applied at the plate surface while N_x and N_y are the membrane forces distributed along the edges of plate and N_{xy} is the in-plane shear force, respectively.

The simplest form of plate vibration analysis represents the free vibration of unloaded rectangular plates with negligible damping. In this case, the forcing terms at the right side and the viscous damping term at the left side of Eq. (1) are set to zero and Eq. (1) reduces into Eq. (2).

$$D\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]^2+\rho h\ddot{w}=0 \quad (2)$$

There is an infinite number of natural frequencies (and mode shapes) available from the closed form or numerical solutions to Eq. (2) for various boundary conditions of rectangular plates [2]. As an example, the closed form solution to Eq. (2) for rectangular plates with all edges simply supported could be presented as Eq. (3).

$$\omega_{m,n}=\pi^2[(m/L)^2+(n/B)^2]\sqrt{D/\rho h} \quad (3)$$

In which m and n indicate the number of wave fronts along the length *(L)* and breadth *(B)* of the rectangular plate for any particular mode shape with natural frequency $\omega_{m,n}$. Amabili [2] verified the natural frequencies of 15 modes of vibration resulted from Eq. (3) experimentally for the case of a rectangular aluminium plate.

In another case, the free vibration of rectangular plates with negligible damping in presence of membrane forces was studied by Singh and Dey [34] using the energy method. They discretized the total energy of free vibration of the system by replacing the derivative terms with their Finite Difference equivalents and used a specific energy minimization technique to solve the resulting eigenvalue problem. One step further into the case of rectangular plate vibration with membrane forces, Leissa and Kang [17] presented one of the first exact solutions for the case of vibration and buckling of rectangular plates with a particular set of boundary conditions in which two opposite edges of the plate were clamped and the two other edges were simply supported. A linearly variable in-plane load was considered to act at the simply supported edges [17]. A similar case was presented by Wang et al. [40] that simplified Eq. (1) into the form presented in Eq. (4) for the case of free vibration of rectangular plates with membrane forces acting at opposite edges along x axis [40].

$$D\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]^2+\rho h\ddot{w}=N_x\frac{\partial^2 w}{\partial x^2} \quad (4)$$

Wang et al. [40] used the differential quadrature method to solve this equation and calculated the vibration frequency for six mode shapes of the rectangular plate (*m = 1–3, n = 1 and 2*). They estimated the *w(x,y,t)* function by *w(x,y)*sin(*ωt*) and presented the in-plane force N_x with a linear function. More recently, Akhavan et al. [1] studied a similar case where closed-form solutions for the free vibration analysis of Mindlin plates with uniform and linearly distributed in-plane loading at two opposite edges with simply supported boundary conditions were

Download English Version:

https://daneshyari.com/en/article/6777531

Download Persian Version:

https://daneshyari.com/article/6777531

Daneshyari.com