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A new analytical solution and novel energy formulations for non-linear eccentric impact analysis of composite multi-layer/sandwich plates resting on point supports

M. Shariyat*, M. Roshanfar

Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Tehran 19991-43344, Iran

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ABSTRACT

In the present research, an analytical solution based on a new idea of superposition of two kinematic descriptions is presented for dynamic response analysis of a multi-layer/sandwich composite plate with point supports subjected to an eccentric low-velocity impact. Direct and virtual-work-based novel energy formulations are proposed for the problem that take into account the potential energy of the indentation region. The nonlinear governing equations of motions are found based on minimization of the total potential energy of the whole mass-plate system, including work of the inertia forces, employing Ritz technique and transformation of the time-dependent nonlinear system of governing equations to a non-linear algebraic one through a novel concept. In contrast to the available researches, influence of the lower layers on the stiffness of the contact region is incorporated. Time-dependent responses of a sandwich composite plate with simply supported edges are compared with those of a plate resting on point supports. Verification of the results has been accomplished based on results of ABAQUS computer code. In the results section, the significant effects of the point supports (in comparison to the complete edge supports), initial velocity of the indenter, aspect ratio of the plate, and material properties of the layers on time histories of the contact force and lateral deflection of the plate are investigated.

1. Introduction

Due to the capability of collecting layers of quite different rigidities, utilizing the multilayer/sandwich structures has become extensively common in the construction of various engineering structures, such as aerospace, mechanical, automotive, marine, offshore, and civil engineering structures. These structures are mainly used for carrying the transverse, bending, or torsional loads. Since in the sandwich plates loads are mainly carried by the upper and lower face sheets, these layers are made of high strength materials. The middle layers or core that are often used to increase the distance between the top and bottom layers and consequently, increasing the moment of inertia of the cross-section, can be manufactured from less expensive and generally lower stiffness materials. However, some researchers have suggested using FGM cores or foams [1–5]. In addition to achieving a more economic structure, the core and the intermediate layers may provide a suitable insulation or structural damping for suppression of the vibration amplitude or unwanted impact energies [6–8]. Despite their high strength, these plates are sensitive to impacts exerted by foreign objects and may encounter various damages such as: separation of layers, fibers or matrix breakage, and slippage of the fibers relative to the matrix. Therefore, recognition of the parameters that affect behaviors of the sandwich plates against impact loads is a vital task.

Majority of the researches performed in the field of low-velocity impact analysis of the plates were mainly focused on composite or functionally graded (FGM) plates with perfect supports [9–19]. Many of the impact investigations have been performed either experimentally or using the commercial finite element analysis codes. The researches of closer relevance to the present work are those performed based on solved formulations and thus, only this type of researches will be introduced here. Palazotto et al. [20] predicted responses of the composite sandwich plates to low-velocity impacts by means of a plate bending finite element algorithm used a fifth order Hermitian interpolation allowed three-dimensional equilibrium integrations for transverse stress calculations. The contact pressure was computed by Hertz law and the contact radius was determined iteratively. Burlati [21] obtained general responses of low-velocity impact of a sandwich plate based on Hertz's contact law. Anderson [22] used single-degree-of-freedom models for large rigid sphere mass impact on composite sandwich laminates. Energy-dissipating elements were incorporated

* Corresponding author.
E-mail addresses: shariyat@kntu.ac.ir (M. Shariyat), majidroshanfar@gmail.com (M. Roshanfar).

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Nomenclature

a	Plate length
A	Area
b	Plate width
A_m, B_m, C_{mn}	Series coefficients
C_{ij}	Elasticity coefficient
D, D_{ij}	Bending rigidity
E	Modulus of elasticity
E_1, E_2	Moduli of elasticity of the bodies in contact
f	Function
$\mathscr{G}_{11}$, $\mathscr{G}_{12}$, $\mathscr{G}_{13}$, $\mathscr{G}_{14}$, $\mathscr{G}_{21}$, $\mathscr{G}_{22}$, $\mathscr{G}_{23}$, $\mathscr{G}_{24}$	Coefficient of the numerical time integration
G	Shear modulus of elasticity
h	Total thickness
h_1, h_2, h_3	Thickness of the face sheets and core
$\mathscr{H}_{11}$, $\mathscr{H}_{21}$	Coefficients of the numerical time integration
$\mathcal{H}_{12}$, $\mathcal{H}_{22}$	Vectors of the numerical time integration
i	Layer number, iteration counter
k	Stiffness of the contact region
$\boldsymbol{K}$	Stiffness matrix
$\hat{\mathscr{K}}$	Augmented stiffness matrix
m, n	Counter
m_i	Mass of the indenter
M_x, M_y	Moments per unit length
$\boldsymbol{M}$	Mass matrix
M,N	Series limits
N	Number of the layers
p	Intensity of the distributed load
P	Impact load
P_m	Peak indentation load
R	Radius of the indenter; effective radius
R_1, R_2	Radii of curvature of the contact surfaces
$\mathcal{R}$	Augmented force matrix
t	Time
V	Initial velocity of the indenter
w	Lateral deflection
x,y,z	Coordinates
X, Y	Functions of the x and y coordinates
α	Indentation magnitude
α_m	Peak indentation magnitude
α_0	Magnitude of the permanent indentation
$\boldsymbol{\delta}$	Vector of the unknown parameters
δ_{ij}	Kronecker's delta
ε	Very small number
$\boldsymbol{\varepsilon}$	Strain vector
ν	Poisson ratio
ν_1, ν_2	Poisson ratios of the bodies in contact
ρ	Mass density
Π	Total potential energy
$\boldsymbol{\sigma}$	Stress vector
Ω	Plate volume

into the models to account for the material damage. A higher-order impact model was presented by Yang and Qiao [23] to simulate responses of a soft-core sandwich beam subjected to a foreign object impact, using a Hertzian contact law. The calculated stresses were used to check the failure occurrence. Icardi and Ferrero [24] presented a finite element simulation for impacts on sandwich composites, based on a finite-element-based zig-zag representation of the displacements. Hertz contact law and Newmark's implicit time integration scheme were used for solving the contact problem and iteratively determination of the contact radius and force. Chai and Zhu [25] presented a brief literature survey on impact analysis of the sandwich plates. Feli et al. [26] presented a mass-spring model for a clamped circular composite sandwich panel with a rigid perfectly plastic core resting on the ground and subjected to impact by a spherical impactor. Natsuki et al. [27] discussed effects of inserting an elastic layer between two layers of a laminated plate on the low-velocity impact responses.

Point supports may be considered deliberately (e.g., supporting of a table by its four legs) or they may be a result of manufacturing errors (so that the plate may rest only on its corners and not all points of the edges, due to the undeliberate initial curvatures). The foregoing literature survey indicates that the impact behavior analyses of the plates were presented based on either numerical or discrete models, in the previous studies. The novelties included in the present research may be summarized as:

(i) Presenting an analytical solution instead of the data-dependent finite element or commercial-FEA-codes-based analyses.
(ii) Considering an eccentric impact.
(iii) Proposing the analytical solution for a plate with point supports and comparing impact responses of plates with full edge supports with those of plates with point supports, even for eccentric impacts. The idea of the proposed analytical solution has several superiorities on the traditional separation of variables approaches.
(iv) Presenting novel energy formulations that in contrast to all the available researches, takes into account the potential energy of the indentation region.
(v) The energy formulations are proposed in direct and virtual-work-based forms.
(vi) Incorporating effect of the stiffness of the lower layers on the contact region, in contrast to the available researches.
(vii) Transformation of the time-dependent nonlinear governing equations to a set of time-independent nonlinear algebraic system of equations.
(viii) Presenting new results for eccentric impact of sandwich plates with point supports

2. The governing analytical equations

2.1. Modification of Hertz contact law to consider stiffness of the lower layers

The geometric and kinematic parameters of the multilayer/sandwich plate on point supports, as well as, the adopted coordinate system are shown in Fig. 1. In this figure, the length, width, and thickness of the plate are represented by a, b, and h, respectively. Origin and x-y plane of the coordinate system are located at the mid-plane of the plate and the x and y axes are coincident with two edges of the plate, as shown in Fig. 2. R and V denote radius and initial velocity of the indenter, respectively. Number of layers of the plate is arbitrary and not

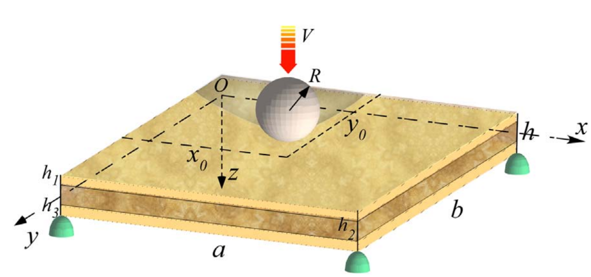

Fig. 1. The geometric and kinematic parameters and the coordinate system of the multilayer/sandwich plate on point supports.