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A force-based method for identifying the deformation modes of thin-walled members

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ABSTRACT

There are three kinds of fundamental deformation of thin-walled members, namely global, distortional and local modes. A set of identification criteria based mainly on the characteristics of forces rather than conventional deformation shapes are proposed. The three newly defined deformation modes based on these criteria cover the entire deformation field of a thin-walled member, including those parts triggered by shear strains and transverse membrane strains. Moreover, it is proved that the three deformation modes are orthogonal to each other with respect to the elastic stiffness, which indicates the accuracy of the corresponding separation of the total strain energy, the portion of which is then designated as the participation factor of the individual mode, which is widely considered to be more physically appealing. Theoretically, the proposed criteria impose no restriction on cross-sectional types or geometric/loading boundary conditions. The proposed criteria are utilised to decompose the buckling results of several examples calculated by Finite Element Method. The resulting participation factors of each buckling mode are compared with those from the base functions of the constrained Finite Stripe Method (cFSM). The comparisons between the results of the two methods indicate the validity of the proposed criteria.

1. Introduction

The stability capacity of a thin-walled member is affected by the physical non-linearity, imperfections and post-buckling strength reserve, and their effects are tied to the nature of the buckling classes, namely the global, distortional and local modes. This demonstrates why the correct identification and classification of the buckling modes, as well as the accurate calculation of the corresponding critical loads, have chief importance in determining the ultimate load-carrying capacity of a thin-walled member [1].

Since general purpose numerical methods, such as the Finite Element Method (FEM) and the Finite Strip Method (FSM), are not able to segregate the various buckling classes [2], presently, the Generalised Beam Theory (GBT) [3,4] and the constrained Finite Strip Method (cFSM) [2,5] are widely used to perform the separation. Meanwhile, researches remain active in the fundamental development of incorporating GBT and/or cFSM into the FEM models.

1.1. Generalised beam theory

The Generalised Beam Theory extended the traditional beam theory to be able to take into account the cross-sectional deformation. In the modal analysis, orthogonal deformation modes are defined by constructing one [3] (or a series of [6]) eigenvalue problem. Note that in all the GBT basic stiffness matrices, including $\boldsymbol{C}$(warping), $\boldsymbol{B}$(transverse), $\boldsymbol{D}$ (shear) stiffness, and coupling stiffness ($\boldsymbol{H}$ [3], $\boldsymbol{F}$ [3] and $\boldsymbol{E}$ [6]), the GBT basic modes are only orthogonal about $\boldsymbol{B}$ and $\boldsymbol{C}$ (or only about $\boldsymbol{C}$ when considering the shear strain [6]), since it is impossible to diagonalise more than two matrices simultaneously [3].

1.2. Constrained finite strip method

In the cFSM, according to three principles [2], the buckling modes of thin-walled members are divided into four deformation mode families, which are the global (G), distortional (D), local (L) and other (O, or ST) modes. Despite the practically negligible limitations inherited from FSM (i.e. prismatic member, limited boundary conditions, etc.) [2], a mass of extensions are undertaken in the domain of the cFSM or inspired by it [5].

As for the orthogonalities among the basic modes, they are more complex than those in GBT. Within each of the G, D, L mode spaces, the orthogonality between each two sub-modes is with respect to the elastic stiffness matrix $\boldsymbol{K}_e$ [2], which stems from a combination of plate bending and membrane plane stress. However, the orthogonality

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Notation and terminology

(1)Basic factors:

- n number of cross-sections (including ending sections),
- k number of *main nodes* on a cross-section,
- p number of *internal main nodes* on a cross-section;

(2)Vectors:

- $\boldsymbol{\Delta}$ displacement vector,
- $\boldsymbol{P}$ load vector;

(3) Classification of the $\boldsymbol{\Delta}$ and $\boldsymbol{P}$ vector components (denoted in subscript form):

- $\bullet_E$ universal set (according to all DOFs),
- $\bullet_{mX}$ translations or forces of the *main nodes* in the X-direction (longitudinal direction),
- $\bullet_{mY}$ those of the *internal main nodes* in the Y-direction,
- $\bullet_{mZ}$ those of the *internal main nodes* in the Z-direction,
- $\bullet_{mTr}$ $\bullet_{mTr} = (\bullet_{mY}) \cup (\bullet_{mZ})$,
- $\bullet_m$ $\bullet_m = (\bullet_{mX}) \cup (\bullet_{mTr})$, the *representative membrane* subset of the universal set (loaded DOFs in the *integrated sub-deformation*, also constrained DOFs in the *local sub-deformation*),
- $\bullet_c$ $\bullet_c = \overline{\bullet_m}$, the *local-plate* subset of the universal set (unloaded DOFs in the *integrated sub-deformation*, also unconstrained DOFs in the *local sub-deformation*);

(4) Decomposition of $\boldsymbol{\Delta}$ and $\boldsymbol{P}$ (denoted in the superscript form):

- $\bullet^O$ $\boldsymbol{\Delta}$ or $\boldsymbol{P}$ of the *original deformation*, $\bullet^O = \bullet^L + \bullet^I$
- $\bullet^L$ those of the *local sub-deformation*,
- $\bullet^I$ those of the *integrated sub-deformation*, $\bullet^I = \bullet^G + \bullet^D + \bullet^R$
- $\bullet^G$ those of the *global sub-deformation*,
- $\bullet^D$ those of the *distortional sub-deformation*,
- $\bullet^R$ those of the *rigid-body sub-displacement*;

(5) Functions for the resultant forces:

- $\mathscr{F}_x, \mathscr{F}_y, \mathscr{F}_z$ cross-sectional resultant X-, Y- and Z- direction forces,
- $\mathscr{M}_x, \mathscr{M}_y, \mathscr{M}_z$ cross-sectional resultant X-, Y- and Z- direction moments,
- $\mathscr{B}$ bimoment on a cross-section.

between the G, D and L mode spaces is actually only with respect to the membrane warping stiffness, as is shown in [8] as

$$\int_A u_i \cdot u_j \mathrm{d}A = 0, \tag{1}$$

where u_i and u_j are axial translations in G and D respectively, and A is the cross-section area. Note different strategies were used to achieve O/ST mode, and multiple orthogonal schemes have been proposed. Please refer to [2,7,8].

1.3. Constrained finite element method (cFEM)

The preliminary attempt of applying the constraining technique for shell FEM was performed in [9], where M Casafont et al. developed a method for the calculation of pure buckling loads by constraining the nodal displacements of the FEM mesh with shell elements to force the model to buckle in the individual GBT deformation modes. Due to the differences of the constitutive relations and element interpolating functions between the GBT and the conventional shell elements, the pure buckling loads would not be the same if the DOF relations in the FEM model were defined in full accordance with the GBT modal definition. An effective, meanwhile somewhat empirical, measure neglecting some constraints in the FEM was introduced, in order to get the resulting critical forces close to GBT.

Recent works [10–12] of cFEM aimed at the derivation and the application of a novel shell finite element based on an unusual combination of otherwise well-known shape functions [10]. The basic mechanical criteria of mode decomposition were essentially similar to cFSM, but the change in longitudinal interpolation significantly extended the practical applicability of the method to a wide range of practical problems [10–12].

1.4. FEM buckling mode identification method

S Ádány [13] implemented the modal identification of the FEM model buckling analysis by applying the modal basis from the cFSM to simulate the FEM buckling analysis results of thin-walled members. A significant computational effort was needed because the simulation was conducted between two different interpolation patterns along the length of a member: segmented low-order interpolation pattern of FEM and orthogonal trigonometric basis of FSM. A process of reducing the number of the cFSM base functions may be needed for the efficiency of this method [13]. It was pointed out that significant errors may be found in the simulate results when the half-wavelength of the FEM buckling mode is shorter than the minimum half-wavelength of the cFSM base functions or close to the FEM element length [13].

M Nedelcu et al. [14,15] formulated a fundamental deformation mode identification method of general buckling modes provided by the finite shell element analysis of isotropic thin-walled members based on the orthogonality of the GBT modes. Regarding the method limitations, the membrane transverse extensions and shear strains are neglected following GBT classical assumptions. For some buckling modes, the errors introduced by the GBT simplifying assumption of linear distribution of the warping displacements along the member cross-section are not negligible [14].

In these two methods, the modal participation factors are determined according to the displacement vector norms of the sub-deformations. The norm of the warping displacements is usually 1 or 2 orders of magnitude smaller than the norm of the transversal displacements [14]. Therefore, strain energy is considered to be more physically appealing [8]. It was also pointed out in [15] that a more refined procedure should involve the computation of the modal participation factor depending on the strain energy produced.

1.5. Research significance – a force-based method

In this paper, a set of mechanical criteria are proposed to divide the entire deformation field of thin-walled members into the global (G), distortional (D), and local (L) deformation mode classes, which are orthogonal to each other with respect to stiffness of the member.

Compared with the FSM or GBT model, there are more DOFs in a FE model with the same discretization of a cross-section, thus it is impossible to transform the whole FE displacement field into a cFSM or GBT modal basis [8]. More basic mode functions compared to those employed in the cFSM or GBT are introduced in this work to completely partition the entire deformation field of the FE model — the first objective of this research.

The second objective of this research is to determine the mode participation factors according to the strain energy produced. It should be noted that only when the basic modes are orthogonal to each other about the stiffness can their strain energies exhibit unambiguous