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A nonlinear resonance (eigenvalue) approach for computation of elastic collapse pressures of harmonically imperfect relatively thin rings

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ABSTRACT

A nonlinear resonance (eigenvalue) based semi-analytical approach is employed here for computation of the elastic mode 2 collapse pressures of moderately-thick to thin isotropic rings, weakened by harmonic or modal type imperfections. The mode 2 collapse pressure is, by definition, associated with the buckled mode shape of cos(2θ) type, and is the harmonically imperfect ring counterpart to the Euler type buckling pressure of a hydrostatically pressurized thin perfect ring. A von Karman type iterative nonlinear analysis, which is based on the assumptions of transverse inextensibility and first-order shear deformation theory (FSDT), is utilized for computation of hydrostatic collapse pressure of a harmonically imperfect ring. Interesting and hitherto unavailable numerical results pertaining to the effects of harmonic imperfections on the hydrostatic collapse pressures of imperfect metallic rings are also presented.

1. Introduction

The buckling response of metallic cylindrical shell/ring type structural components is of great concern to submarine structural designers. Of primary interest to designers and operators alike is the sensitivity of the buckling response of ring or very long cylindrical shell type structures to geometrical defects, such as modal imperfections, e.g., out-of-roundness, which are apt to be generated during a large scale fabrication process of such submersibles. Investigation of the influence of this type of defects on the stability (or lack thereof) of ring/shell type structural elements is the primary objective of the present study.

Buckling and postbuckling responses of ring type structures have been extensively studied in the literature [1–9]. The elastic buckling pressure of a ring is typically associated with the buckled mode shape of cos(2θ) type, termed here mode 2. The critical pressure of a perfect ring is typically computed by using a linear (linearized) eigenvalue analysis, associated with the conventional bifurcation theory [1,7]. However, structural imperfections, which are expected to be always present, prevent a structure from reaching its predicted Euler type buckling pressure. Nonlinear buckling (nonlinear eigenvalue) analysis is more general and accurate than its linear (eigenvalue) counterpart, because this permits incorporating geometric imperfections, load perturbations, etc., into the formulation [10,11]. The differences between the solutions of linear and nonlinear eigenvalue problems are clearly depicted by Keller and Antman [12] in Figs. 1 and 2 of the chapter “Introduction” of Ref. [12]. For example, (i) the branches of the nonlinear eigen solution, emanating from the eigenvalues of the linear problem, may be curved, (ii) there may be no branch of the nonlinear solution emanating from the eigenvalue(s) of its linearized counterpart, (iii) there may be several branches emanating from a single eigenvalue of the linearized problem, (iv) there may be a secondary bifurcation, (v) the branches from distinct eigenvalues of the linearized problem may be connected, or (vi) there may be branches of solution in nonlinear eigenvalue problems that do not emanate from the eigenvalues of the linearized problem, such as the branch C of Fig. 2 of Ref. [12].

The mathematical modeling involves increasing the applied load (pressure) in small increments until the buckled shape of the ring associated with the mode 2 becomes unstable (i.e., suddenly, a very small increment in applied pressure will cause a very large deflection associated with this mode) [10,11]. It may be remarked here that since the elastic postbuckling of a ring involves deformation hardening type nonlinearity [3,4,7,13,14], there is no final loss of stability in the postbuckling stage, but buckled shape is no longer of the mode 2 type. This is in stark contrast to the deformation softening type nonlinearity caused by, e.g., the thickness effect, [15–18], presence of distributed fiber misalignments (in fiber reinforced composites) [18,19] and hypoelastic or inelastic material properties [13,14,20] in addition to the thickness effect [21–25]. It may be noted in this context that Tvergaard and Needleman [26], in relation to the compression failure of thin metallic structures, such as an elastic column on a softening (inelastic) foundation and an elastic-plastic plate, were among the first to observe that the applied load-deflection curve attains a maximum or limit load point, and also "the final buckled configuration involves a localized buckling pattern, in contrast with the periodic deformation pattern

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Nomenclature[1]

${}_{0}C_{ijrs}$	incremental elastic stiffness (material property) tensor
${}^{t+\Delta t}ds$	differential loading surface arc length evaluated at the first iteration of each load step when hydrostatic pressure is applied
${}^{0}dA$	infinitesimal control area of a ring with respect to the initial configuration
E, G	Young's and shear moduli of an isotropic material
${}_{0}\bar{e}_{ij}^{L}$	linear incremental component of the strain tensor
${}_{0}\bar{e}_{ij}^{N}$	linearized incremental component of the strain tensor
$F_i(u,\phi,w_1,w_2)$	component of the nonlinear force vector, for i = 1,2,3,4
F_i	component of the linear force vector, for i = 1,2,3,4
$\{{}^{t+\Delta t}\mathbf{f}_{L}\}$	applied load vector at time t + Δt
$\{{}^{t+\Delta t}_{0}\mathbf{f}_{NL}\}$	nonlinear internal force vector at time t + Δt
g_{φ}	coefficient of the first fundamental differential quadratic form of the middle surface (center line) of an imperfect ring in the φ direction
h	thickness of the ring
$[{}^{t}_{0}\mathbf{K}_{L}]$	linear global stiffness matrix
$[{}^{t}_{0}\mathbf{K}_{NL}]$	nonlinear contribution to the global geometric stiffness matrix
p	uniform hydrostatic pressure
p_{cr}	hydrostatic buckling pressure of a perfect ring
R, R_i	radius of curvature of the middle surface (center line) and inner surface, respectively, of a perfect ring
${}^{t+\Delta t}R$	external virtual work done on a two-dimensional (ring) body
R_1	approximate radius of curvature of the middle surface (center line) of an imperfect ring
r(θ)	radial coordinate of the middle surface (center line) of an undeformed ring with modal imperfection
${}^{t+\Delta t}_{0}S_{ij}$	second Piola-Kirchhoff stress tensor at time t + Δt evaluated with respect to the initial configuration
${}^{t+\Delta t}s$	loading surface arc length evaluated at the first iteration of each load step when hydrostatic pressure is applied
t	time as an index
${}^{t}u$	circumferential displacement component at time t
Δu	incremental circumferential displacement component during the time step from t to t + Δt
ΔW	incremental virtual work during the time step from t to t + Δt
${}^{t}w_1$, ${}^{t}w_2$	linear and nonlinear contributions to the transverse displacement component (deflection) at time t
Δw_1, Δw_2	linear and nonlinear contributions to the incremental transverse displacement component (deflection) during the time step from t to t + Δt
X, Z	cartesian coordinates of a point on the inner surface of a deformed ring
x, z	cartesian coordinates of a point on the middle surface (center line) of an undeformed ring
ε	amplitude of modal imperfection (eccentricity) of a ring
ε_{φ}	extensional strain component in the direction φ
$\varepsilon_{\varphi\zeta}$	shear strain component in the φ-ζ plane
${}^{t}\phi$	rotation of the normal to the middle surface (center line) of the deformed ring at time t
$\Delta\phi$	incremental rotation of the normal to the middle surface (center line) of the deformed ring during the time step from t to t + Δt
φ, ζ	coordinates attached to the middle surface (center line) of an imperfect ring
θ	angle measured from the global x axis
σ_{φ}	extensional stress component in the direction φ
$\sigma_{\varphi\zeta}$	shear stress component in the φ-ζ plane

associated with the critical buckling mode".

A review of the literature suggests that since the pioneering work of Euler in the eighteenth century, who first derived the buckling load of an initially straight isotropic column, that has been utilized by engineers in column design with appropriate factors of safety, no corresponding analysis has been made available to engineers for computing the "buckling" load of an initially imperfect isotropic column. The analysis of a perfect ring is analogous to its counterpart for an initially straight column, and a similar conceptual gap existed until recently in regards to the analysis of a modally (or harmonically) imperfect ring. The primary objective of the present investigation is to present a nonlinear resonance (eigenvalue) based semi-analytical solution technique for prediction of the elastic mode 2 collapse pressures of moderately thick rings, weakened by modal or harmonic type imperfections to bridge this centuries-old conceptual gap, and to provide engineers a computational tool for the prediction of buckling/collapse pressure of an imperfect ring. A von Karman type iterative nonlinear analysis, which is based on the assumptions of transverse inextensibility and first-order shear deformation theory (FSDT), is employed for computation of hydrostatic collapse pressure of the imperfect ring.

In what follows, the theoretical formulation of a modally imperfect ring is described in Section 2. In Section 3, the collapse pressure is computed by employing an incremental solution approach starting from the method of virtual work and the total Lagrangian (TL) formulation that results in linearization of the non-linear strain-displacement relations in terms of the incremental displacement components including rotation. A general discussion on the method of virtual work and linearized equations of motion by total Lagrangian formulation is provided in Appendix A, while Appendix B briefly describes the linearized buckling analysis of a perfect ring, based on the (linearized) bifurcation theory. The Newton-Raphson iteration scheme is implemented to solve the resulting matrix equations (see Appendix C). Double symmetry conditions permit the model to be limited to a quarter of the ring. The assumed solution functions for incremental displacement components satisfy the prescribed hoop boundary conditions a priori, and unlike a finite element analysis (FEA), no discretization of the ring geometry is necessary, which saves a substantial amount of computation time. Finally, hitherto unavailable numerical results pertaining to the effects of harmonic imperfections on the hydrostatic collapse pressures of thin metallic (isotropic) rings are also presented.

2. Theoretical formulation

2.1. Differential geometry

Figs. 1 and 2 show schematics of a perfect and an imperfect ring, respectively. Fig. 3 exhibits the geometry of the middle surface (center line) of the same imperfect ring. Here, (φ,ζ) represents the orthogonal curvilinear coordinates such that the φ denotes the line of curvature coordinate of an imperfect ring (with mode 2 imperfection) at the midsurface, while θ represents the corresponding line of curvature of a perfect ring, and ζ is a straight line normal to φ. The principal curvature of the reference surface coincides with that of the coordinate line, φ, and the value of the principal radius of curvature is denoted by R_1. The principal radius of curvature of the corresponding perfect ring is denoted by R.

The mean radius of an imperfect ring can be represented as $r = R + \varepsilon\cos 2\theta$, where ε is the amplitude of modal imperfection of the ring. As

[1] Please note: A bold symbol denotes either a vector or a matrix.

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