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Coupled bending and torsional vibrations of non-uniform thin-walled beams by the transfer differential transform method and experiments

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ABSTRACT

This paper describes a new methodology capable of analyzing coupled bending and torsional vibrations in non-uniform thin-walled beams. The transfer matrix method (TMM) and the differential transform method (DTM) are combined to create the proposed transfer differential transform method (TDTM) for solving equations coupling bending and torsional vibrations in non-uniform beams. Compared with the finite element method (FEM), the TDTM utilizes a type of changeable mode shape functions so that the number of meshed elements can be reduced greatly when the beam's geometric size is uniform. The equations of motion of the non-uniform thin-walled beams are established using Hamilton's principle. The method considers both the warping and rotary effects of the beam section. The natural frequencies and mode shapes of the bending and torsional components are obtained using the TDTM. The accuracy of solutions can be controlled by the mesh density and the series expansion order of the mode shape whose ranges are also discussed. For illustrative purposes, the natural frequencies and the frequency response curves of a uniform beam and a non-uniform beam are studied respectively and are validated by experiments. The effects of warping and the distance between the centroid of a given section and the corresponding shear center on the vibration properties of the beam are also investigated.

1. Introduction

Non-uniform thin-walled beam can be widely found in blade structures and many other engineering applications. If the blade has a long span, such as in the case of a helicopter blade or a wind turbine blade, it is reasonable to simplify the structure as a non-uniform beam. Besides, in each blade section, the shear center may not be coincident with its centroid, which results in a coupled bending and torsional effect. There have been several papers examining the dynamic features of such a model. Most of them used the conventional methods while dealing with the associated partial differential equations, such as the finite element method (FEM), the dynamic stiffness matrix method and the Ritz method. In this paper, the transfer differential transform method (TDTM) is applied to solve the coupled partial differential equations. The method greatly reduces the mesh number and improves the computing efficiency by combining the advantages of the transfer matrix method (TMM) and the differential transform method (DTM).

Problems arising from coupling of bending and torsion in open section beams have been examined by several researchers. Mei [1] used the FEM to study channel beam vibration problems and compared the computing results with the experimental results of the first-order frequency. Dokumaci [2] solved the equations of motion without considering the warping effects.

Banerjee [3] and Banerjee and Williams [4] extended Dokumaci's theory, and developed the dynamic matrix methods for solving equations coupling bending and torsion, while taking the influences of shear deformation and axial loads into account. However, the warping effects were neglected. Lai et al. [5] studied the dynamic properties of a C-shape composite channel beam. The material properties needed for the formulation were obtained using the law of average and the warping stiffness was considered. Ritz method was used to formulate the accompanying eigenvalue problem. Bishop et al. [6] demonstrated that this approach would lead to considerable errors while solving the dynamic characteristics if the warping strain energy term is omitted. Klausbruckner and Pryputniewicz [7] employed the FEM for studying coupled bending and torsion. The warping stiffness was considered in the equations of motion. The errors between the experimental and the numerical results were analyzed.

The coupled bending and torsional vibrations in two directions of a thin-walled beam were investigated by Tanaka and Bercin [8]. Li et al.

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[9,10] reviewed the works of former researchers and gave two illustrative examples of thin-walled beams with a single symmetric axis. The natural frequencies and the mode shapes were calculated by the dynamic TMM. The effects of axial force, warping stiffness and shear deformation were also included. Ozgumus and Kaya [11,12] employed the DTM for solving a beam with linearly changing cross section. Hamilton's principle was applied to derive the energy expressions. The high efficiency resulting from the DTM while solving the coupled differential equations was demonstrated. However, the section of the beam was assumed to be solid and the warping stiffness was not considered.

Based on the Vlasov's theory, Borbon et al. [13] established the equations of motion of a non-symmetric thin-walled beam subjected to an axial load. Shear deformation and rotatory inertia were considered. Egidio et al. [14] studied the nonlinear warping effects of a thin-walled beam with open cross section. Galerkin's discretization was performed by using a suitable expansion of displacements based on the mode shapes. The dynamic responses of the beam subjected to a harmonic force were investigated. Li et al. [15,16] used the Ritz method to solve the forced vibration problems arising in beams and shells and investigated the frequency response curves.

Until recent years, Ozgumus and Kaya [17] conducted an extended analysis of a rotating Timoshenko beam with the bending-bending-torsion coupling effect by using the DTM. Bastawrous and EI-Badawy [18] used the assumed modes method to discretize the equations of motion and studied the coupled bending and torsional vibrations of the wind turbine blades. Rao and Jin [19] focused on uncertainties associated with the problem of coupled bending and torsion in beams. They compared the results obtained from a truncation-based interval analysis method, a universal grey number-based approach and an interval-discretization method, respectively. Liu and Shu [20] investigated the coupled bending-torsion vibration of a single delamination beam subjected to axial loads and static end moments. The buckling loads and critical moments were also calculated. Similarly, the analytical results were compared with those obtained from some commercial software and with the experimental data by Kashani et al. [21]. Axial loads and end moments were considered while performing the beam stability analysis. Li et al. [22] analyzed the stochastic responses of an axially loaded composite beam with bending-torsion coupling considering the effects of shear deformation and rotary inertia. Cooley and Parker [23] derived the equations of motion for a spinning cantilever beam to analyze the bending in the direction of rotation axis coupled with the torsional motion through the gyroscopic terms. The natural frequencies and vibration modes were also investigated. Li et al. [24] studied the coupled bending-torsional vibrations of a symmetric laminated composite beam. The dynamic stiffness matrix was formulated from the exact analytical solutions. Bekir and Omer [25] investigated the buckling problem of linear tapered micro-columns using the strain gradient elasticity theory. The results were compared with the modified couple stress theories. Eken and Kaya [26] applied the thin-walled beam theory for an arbitrary cross-section composite beam. The influences of geometrical aspects were studied. Garcea et al. [27] reviewed the Generalized Eigenvectors and the Generalized Beam Theory. They investigated the cross-section deformation modes of thin-walled members with deformable cross-section. Both the calculating efficiency and the accuracy of two approaches were compared in detail. Latalski et al. [28] discussed the dynamic characteristics of a rotating composite thin-walled beam attached with a rigid hub. The differential equations of motion featuring bending-twist elastic coupling were derived from Hamilton's principle. A shift of resonance zones and vibration absorption were observed. After that, the strong non-linear torsional angle was taken into account by Bourihane et al. [29]. They used the Asymptotic Numerical Method to solve the non-linear equations. Pavazza et al. [30,31] combined the Vlasov's theory and the Timoshenko's beam bending theory together to derive a novel theory of bending of thin-walled beams with influence of shear. The close-form analytical results were obtained for three-dimensional expressions of normal and shear stresses. Furthermore, the results were compared with the Euler-Bernoulli's beam theory and the FEM.

The discretizing process and the methods for solving the associated partial differential equations have also been developed. The DTM was used to solve the non-linear equations by Wang et al. [32]. Shabnam and Reza [33] extended the application of the DTM to formulate a two-dimensional model that could deal with thin plates with variable thickness. Rajasekaran and Tochaei [34] made some improvements by developing a new method called the differential transformation element method and achieved a satisfactory solution. Nuttawit and Arisara [35] used the DTM to simultaneously solve linear and non-linear vibration responses of elastically restrained beams. Nuttawit and Jarruwat [36] studied the vibration characteristics of a stepped beam made of functionally graded materials via the DTM. Later, a vibration analysis of a thick nano-beam using the DTM was presented by Farzad and Parisa [37]. Mustapha and Hawwa [38] obtained the approximate solutions for a functionally graded micro-scale beam using the DTM. Certain non-classical boundary conditions were included in the analysis. Ravi and Hariharan [39] investigated the vibration characteristics of a fluid conveying single walled carbon nanotube using the DTM.

Although a number of researchers have investigated vibration problems associated with thin-walled channel beams, only a few have validated their computing results against experimental findings. Earlier literatures focused mostly on conventional methods based on the partial differential equations. By contrast, in this work, the analytical results are compared with the experimental ones. The TDTM is developed to solve the vibration problems of a non-uniformed thin-walled beam. The equations of motion of the non-uniform thin-walled beams are established using Hamilton's principle. The coupled bending and torsional vibration equations describing uniform and non-uniform beams are solved respectively. Two illustrative examples are carried out to validate the results of the TDTM by comparing with the experimental data as well as the commercial codes.

2. Equation of motion of the axial symmetric thin-walled beam

The displacements of a reference point on the cross section of a thin-walled beam are shown in Fig. 1. The coupled bending and torsional deformations of the beam can be divided into two independent steps, i.e. the pure torsion with respect to the shear center and the pure bending based on the former torsional deformation. For the first step, the cross section rotates around the shear center in the cross-section view as shown in Fig. 1(a). For the second step, the position of the reference point will be corrected after considering the bending effects in a side view as shown in Fig. 1(b). The points s and c represent the positions of the shear center and the centroid. The symbol e refers to the distance between them. The point P indicates the position after the deflection in the beam cross section while the initial position is expressed as P_0. ψ and w denote the torsional angle and the deflection of the beam. They are functions of the geometrical parameters and the time. Although the warping displacement is not shown in Fig. 1, the potential energy of this part can be directly added into the total potential energy according to the Vlasov's theory [13]. The warping kinetic energy is not taken into account in this work for it is trivial.

The coordinates of the reference point before and after the deformations can be written as (x_0, y_0, z_0) and (x_1, y_1, z_1). The initial position of the reference point is

$$x_0 = x, \tag{1a}$$

$$y_0 = y, \tag{1b}$$

$$z_0 = z. \tag{1c}$$

The deformed position can be expressed as

$$x_1 = x - (z\cos(\psi) + y\sin(\psi))\sin(w'), \tag{2a}$$