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Harmonic response analysis of coupled plate structures using the dynamic stiffness method

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ABSTRACT

In this study, the dynamic stiffness method (DSM) is applied to study the free and forced vibration behaviors of thin, three-dimensionally coupled plate structures. Both the flexural and in-plane vibrations are taken into consideration. In the formulation, the coupled plate structures are divided into several sub-plates, then the dynamic stiffness matrices of the sub-plates are derived separately by combining the superposition method with the projection method. According to the geometrical coupled condition between the sub-plates, the dynamic stiffness matrix of the whole coupled structure is assembled. To validate the present method, three numerical examples of coupled plate structures subjected to external excitations are performed and analyzed. The accuracy and reliability of the present method are demonstrated by comparing the present results with those obtained by the FEM. Then, the effects of the directions of external harmonic excitations and the coupled angle between the coupled plates on the dynamic responses are investigated. Furthermore, a computational efficiency study is given to illustrate the convergence and accuracy of the DSM.

1. Introduction

Thin plate structures are widely encountered in diverse industrial designs, such as naval hulls or aircraft wings. Generally, the plates are always coupled with each other to constitute a complete structure. In practical engineering, plate structures are inevitably subjected to unexpected external excitations, which may result in excessive vibration or noise. Therefore, it is of great significance to study the vibration characteristics of the plate structures in the design process. For many decades, the development of accurate plate theories has received much attention and great efforts have been focused on this field by many researchers [1–4]. For instance, Leissa gave a detailed literature review on this subject in a series of review articles [5–9].

In recent decades, the FEM has been developed and used to solve the vibration problem both in academic field and engineering practice. Though the numerical method is a powerful tool for solving vibration problems of various structures in the low frequency range, an analytical method has better application prospect due to its good convergence and high efficiency. Thus, developing accurate and efficient analytical methods not only has theoretical value but also has great practical significance in engineering. It has always been a research hotspot to seek a fast and efficient analytical method to solve vibration problems of plate structures. Since the flexural vibration plays a dominant role particularly in the low frequency range, a majority of researches have focused on the flexural vibration of plates. Maury et al. [10,11] used a wavenumber approach to study both the vibration response and sound radiation of a randomly excited plate. Liew and his coauthors [12–14] worked out substantive works on plate vibration by applying the differential quadrature method (DQM). Zhou [15,16] used static beam functions as the basic function in the Rayleigh-Ritz method to study the vibrational characteristics of the thin, isotropic rectangular plate with general or elastic boundaries. Lee [17] proposed a spectral element method (SEM) to calculate the dynamic response of the plate subjected to distributed loads, in which the original plate problem was transformed into an effective beam problem with the aid of fast Fourier transform. The wave based method (WBM) [18–20] is a newly developed prediction technique for vibration analysis of plate and shell. In this method the field variables are expressed in terms of global wave functions, which exactly satisfy the governing equation, then the system matrix is obtained based on an indirect Trefftz method. By using the modified Fourier series method, Li and his coauthors [21,22] studied the vibration characteristic of plate structures with general boundary conditions by introducing artificial boundary springs to simulate boundary constrains.

With intensive research on plate theory, some studies [23,24] have shown that in-plane vibration also plays an important role in the dynamic characteristic of plate structure especially in high frequency range. For this reason, many researchers have devoted efforts to the in-

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plane vibration of rectangular plates. Wang and Wereley [25] solved the modal frequency and mode shapes of in-plane vibration of rectangular plate analytically based on the Kantorovich-Krylov vibrational method. Bardell et al. [26] employed the Rayleigh-Ritz method to investigate the in-plane vibration of rectangular plates with various boundary conditions. Dozio [27] presented a reasonably comprehensive set of numerical results for the in-plane free vibration of single-layer generally orthotropic and symmetrically laminated rectangular plate with general boundary conditions based on the Ritz method. Zhang et al. [28] developed a two-dimensional improved Fourier cosine series method to research the in-plane vibration of isotropic rectangular plate with elastically restrained edges, in which arbitrary boundary conditions can be achieved by varying the stiffness of the boundary springs along each edge. Xing and Liu [29] employed the separation of variables method to derive the exact solution of the free in-plane vibrations of rectangular plates for which at least two opposite edges are simply supported. Later, Xing and Liu [30] presented exact solutions of free in-plane vibrations of orthotropic rectangular plates using the same method. Farag and Pan [31] developed a mathematical model to study the free and forced in-plane vibrations of rectangular plate with all four edges clamped. Kang and Shim [32] derived exact solutions for the free in-plane vibration of rectangular plate at least two opposite edges are simply supported by using the classical power series method of Frobenius. Gorman [33–35] employed the superposition method to carry out the free in-plane vibration analysis of rectangular plate with elastic boundaries. The elastic boundary restrains are considered to be uniformly loaded on the boundary lines and symmetrically loaded with respect to the coordinate axis.

However, most of the above methods are limited to single plate structure and cannot be applied to the coupled plate structures. When coupled plate structures are considered, both the flexural and in-plane motions should be taken into account. From the literature review, only a few publications on the vibration of coupled rectangular plates have been reported due to the increased mathematical complexity in contrast with single plate structure. Vergote et al. [36] analyzed the dynamic characteristic of three-dimensionally flat plate assemblies based on the wave based method. Chen et al. [37,38] investigated the vibration characteristics and power transmission of coupled rectangular plate with general boundary conditions. In this method, the unit plates are coupled by artificial coupling spring with arbitrary angle. Ma et al. [39] researched the mid-frequency vibration of coupled plate structure by applying a hybrid analytical wave and finite element method. Wang [40] used the method of reverberation-ray matrix to perform vibration analysis of finite coupled Mindlin plates with a blocking mass at the coupled edge.

In recent years, there is a well-developed strong-form based general method named as the dynamic stiffness method (DSM) which has been widely used in vibration problems of plate structures [41–53]. Among them, Liu and Banerjee [45–51] promoted a novel spectral dynamic stiffness method (S-DSM) to study free vibration of rectangular plate with various boundary conditions and material properties, which combine the merits of both the DSM and the spectral method (SM). In the S-DSM, all the boundary forces and displacements are represented by infinite modified Fourier series, then the frequency dependent dynamic stiffness matrix has been derived by constructing the force-displacement relationship on the plate boundaries. Nefovska-Danilovic et al. [52,53] developed the DSM to investigate vibration problem of plate structures by combining the Gorman's superposition method with the projection method [54]. First, the general solution of the governing equations of motion is obtained by using the Gorman's superposition method. Then the relationship between the forces and displacements along the boundaries has been developed by using the projection method. Consequently, the dynamic stiffness matrix of the rectangular plate can be obtained. Subsequently, Nefovska-Danilovic et al. extended the method to the vibration analysis of plates based on the first-order and higher-order shear deformation theory [55–59]. To the authors' best knowledge, only a few researchers [60–64] have applied the DSM to investigate the vibration behaviors of three-dimensionally coupled plate structures, whereas the DSM was mainly used to study vibration problems of single plate structure or two-dimensional plate assemblies. However, three-dimensionally coupled plates are often used in practical engineering where the plates are assembled to built-up structures, such as ship hulls, fuselages of aircraft and so on. Although the DSM has been used to study the free vibration characteristics of coupled plate structures in some publications [60–62], the plates were restricted to Levy-type. Recently, Damnjanovic et al. [63,64] expanded the DSM to analysis the free vibration characteristics of three-dimensionally coupled plates with various boundary conditions using first-order and high-order shear deformation theories, but the coupled angles between sub-plates are always fixed at $\pi/2$. The main purpose of this paper is to extend the DSM [52] to investigate the harmonic responses of three-dimensionally coupled plate structures with arbitrary coupled angle and general boundary conditions.

The current paper is organized as follows. In Section 2, the dynamic stiffness matrices of out-of-plane and in-plane vibrations are formulated separately based on the combination of the superposition method and projection method. For convenience of matrix coupling, all the dynamic stiffness matrices of unit sub-plates are transformed into global coordinate system with the aid of a space transformation matrix. Then the whole matrix of the coupled plate structure can be obtained by assembling all the dynamic stiffness matrices of unit plates in global coordinate system. In Section 3, solutions of three common coupled plate structures computed by the present method are validated by comparing with those obtained by the FEM software. The effects of coupled angle, the direction of external force and damping on the dynamic responses are investigated. A computational efficiency study is given to validate the accuracy and efficiency of the present method. Finally, the conclusions of this paper are reported in Section 4.

2. Theoretical modeling

Fig. 1 shows the schematic diagram of a general coupled plate structure. Two individual plates are coupled at an arbitrary angle θ. Plate 1 is located in local coordinate system (x_1, y_1, z_1) and plate 2 is located in local coordinate system (x_2, y_2, z_2). The dimensions of the single plate structure are assumed as $2a \times 2b \times h$. In the formulation, the plate is supposed to be homogeneous, isotropic and elastic and its thickness is uniform and quite small compared to the other dimensions

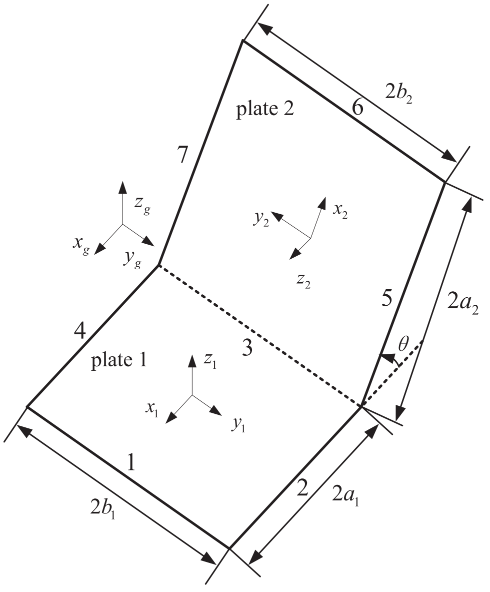

Fig. 1. Two rectangular plates coupled at an angle θ.