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Determination of load paths in plates and shells

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ABSTRACT

Load paths represent the load flow in structures and are used to identify how the load is transferred throughout the structure from points of application to support. The load function method is used in this paper for identification and visualization of load paths in plate and shell structures. Authors previously proposed a load function method to calculate the load paths in plane elasticity problems. This work extends the previous formulation to plates and shells. The significant difference from previous work is that the plates and shells equilibrium equations are not divergence free. Therefore, Helmholtz decomposition is used to decompose the stress field to divergence and curl free components to calculate the load function and find load paths. Mathematical formulations are presented to support the proposed method. Numerical and analytical examples show the application of this method.

1. Introduction

Understanding how load flows through a structure can provide valuable knowledge about the performance and efficiency of the structure and could provide an additional tool to measure the structural functionality of a design. The various load path identification methods have their own definitions and characterizations of "load paths". In this work, load paths are defined as curves that bound regions of constant load flow in a structure.

Initial work on the theory of load paths sought to utilize major and minor principle stress angles as a means of describing the direction of load flow through the structure [1]. However, load paths and principle stress angles vary in definitions. Firstly, principle stress angles describe the angle at which no shear exists in an element. For an area of the structure with high shear stresses, the difference between a vector tangent to the load path and the principle stress angle could be as much as 45 degrees [2]. Secondly, principle stress angles cannot be used to describe a path of constant load flow, as they only represent direction at localized points and give no information on the amount of load carried in a particular region.

Kelly *et al.* introduced stress pointing vectors in the dominant and complementary directions using stress resultants [3,4]. Load paths are defined as tubes of constant force bounded by contours with variable lateral spacing. Based on this definition, the equilibrium condition implies that the normal and shear stresses tangent to the boundary of the path do not contribute to the overall equilibrium in the x-direction [5]. Further work presented examples of the application of load paths to topology optimization [6].

Takahashi [7] and Sakurai et al. [8] presented methods to determine internal load transfer by finding the change of compliance energy inside a structure. The initial strain energy at each node is found using the displacement method. By sequentially constraining individual nodes, then enforcing the same displacements that were found initially, new strain energies can be found at specific locations. The change in compliance energy at a point is the difference between the new strain energy and the original strain energy found at that location. The load path can be found by taking the gradient of the compliance energy scalar field then finding the resulting contour with the smallest gradient. Recently, this method has been extended to rotational and translational six degrees of freedom [9], and orthotropic composites and nonlinear materials [10].

Harasaki and Arora introduced the concept of load transfer and potential load transfer to determine load flow through a structure [11]. For a system of connected elements subjected to applied loads, the load transfer is determined by first finding the displacements and corresponding reaction forces for the structure. Then, by setting the stiffness of the element in question to zero and applying the same displacements found initially, a new set of reaction forces can be found. By taking the difference in reaction forces, the load transfer through the unstiffened element is calculated. This process is repeated for each element until the load transfer in all the elements is determined. Potential load transfer is a similar concept, however it also measures the effectiveness of applying additional stiffness to the structure.

Experimental tests have been undertaken that use load path

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Nomenclature

N_x	Normal stress resultant in x direction
N_y	Normal stress resultant in y direction
N_{xy}	In plane shear stress resultant
Q_x	Transverse shear stress resultant
Q_y	Transverse shear stress resultant
p_x	Pressure in x direction
p_y	Pressure in y direction
p_z	Pressure in z direction
Ψ_x	Load path function of x direction
Ψ_y	Load path function of y direction
Ψ_z	Load path function of z direction
ϕ_z	Potential function for flat plates in bending

identification to map continuous fiber reinforcements onto composite laminates. Li et al. used this methodology to align individual fibers along the load path trajectories of a bolted composite joint [12]. Experimental testing showed a 33% increase in ultimate failure strength and a twofold increase in joint efficiency. Tosh and Kelly performed tests on open-hole and pin-loaded laminates manufactured with trajectorial fiber steering [13]. Fibers were mapped based on principle stress angles, load path trajectories, and a hybrid method that combined both aforementioned methods. Using load path trajectories resulted in an increase in failure loads and outperformed the laminates mapped using principle stress angles.

The authors introduced a load function method to determine load paths, and load flows between them in plane elasticity problems [14,15]. An implicit function called a load function is introduced which its level sets represents the load paths. Load flow between load paths can be determined by subtracting the value of the load function on consecutive load paths. The load function is derived from satisfying the equilibrium and compatibility equations simultaneously. For the case of equilibrium equations with in-plane and transverse loads, such as those for the plate and shell problems, the loads need to be decomposed to curl free and divergence free fields using Helmholtz's decomposition [16–20]. Subsequently, the load function can be derived from the divergence free load field. The proposed formulation leads to load paths in x and y directions and the introduction of the z-direction load path function derived from the z-direction equilibrium of the transverse shear resultants

2. Methodology

In the authors' previous work, the load function formulation is presented for two-dimensional structures loaded in a state of plane stress and is derived from the equilibrium equations in terms of stresses. The stresses are written in terms of the load functions, and the load flow is calculated using the load function level sets. However, for problems with transverse loads in which the stress varies with thickness, multiple load paths through the thickness exist. This problem is remedied by writing the equilibrium equations based on the stress resultant equilibrium equations, as in Eq. (1).

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + p_x = 0$$
$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + p_y = 0$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p_z = 0 \quad (1)$$

where N_x, N_{xy}, and N_y, are in plane stress resultants, Q_x and Q_y are transverse shear stress resultants, and p_x, p_y,and p_z are applied tractions on the surface in x, y, and z directions [21]. Next, the load field with respect to the rectangular frame is defined as follows:

$$\boldsymbol{R_1} = N_x i + N_{xy} j$$
$$\boldsymbol{R_2} = N_{xy} i + N_y j$$
$$\boldsymbol{R_3} = Q_x i + Q_y j \quad (2)$$

Using this definition, the equilibrium equations can be written as:

$$\nabla.\boldsymbol{R} + \boldsymbol{p} = 0 \quad (3)$$

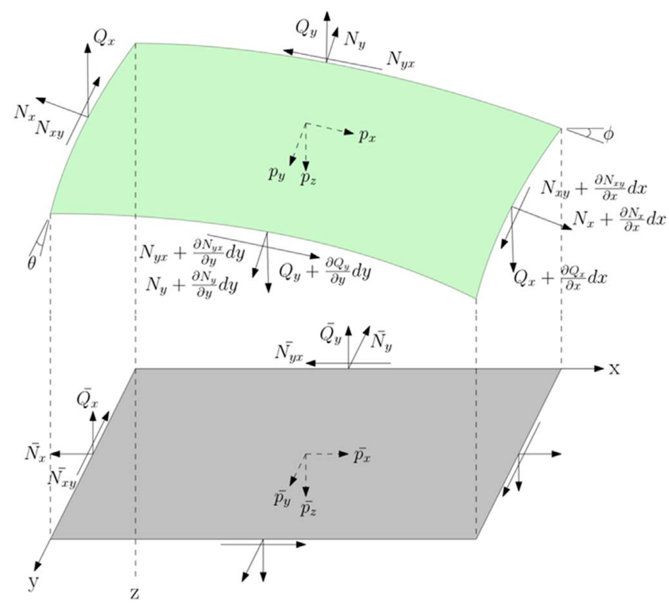

Fig. 1. The projection of shell forces onto the x-y plane.

where $\boldsymbol{R} = [\boldsymbol{R_1}\boldsymbol{R_2}\boldsymbol{R_3}]$, and $\boldsymbol{p} = [p_x p_y p_z]$. The load vector field is decomposed into divergence-free and curl-free components using the Helmholtz decomposition:

$$\boldsymbol{R} = \nabla\boldsymbol{\Phi} + \nabla \times \boldsymbol{\psi} \quad (4)$$

In Eq. (4) the first component ($\nabla\boldsymbol{\Phi}$) is the irrotational component ($\boldsymbol{R^b}$), and the second component ($\nabla\times\boldsymbol{\psi}$) is the self-equilibrated or solenoidal component ($\boldsymbol{R^s}$). The solenoidal vector field admits load functions ($\boldsymbol{\psi} = [\Psi_x \Psi_y \Psi_z]$) and accompanies load paths. The changes in $\boldsymbol{\psi}$ between its two paths equals to the constant load flow of the totally self-equilibrated stress resultants $\Delta\boldsymbol{R^s}$ that is transferred between those two paths. Using the Helmholtz decomposition, first the divergence-free component ($\boldsymbol{R^s}$) is solved for the given boundary conditions and then the curl-free component is determined as the residual ($\boldsymbol{R^b} = \boldsymbol{R} - \boldsymbol{R^s}$). Given a stress resultant field, Eq. (4) can be written as:

$$\nabla \times \boldsymbol{R} = -\Delta\boldsymbol{\psi} \quad (5)$$

and the boundary condition associated with Eq. (5) is:

$$\frac{\partial\boldsymbol{\psi}}{\partial n} = -\boldsymbol{R} \times \boldsymbol{n} \quad (6)$$

The field lines of solenoidal components ($\boldsymbol{R^s}$) are the level sets of $\boldsymbol{\psi}$. After obtaining $\boldsymbol{\psi}$, then solenoidal components can be written as:

$$\boldsymbol{R^s} = \nabla \times \boldsymbol{\psi} \quad (7)$$

then by using the irrotational components ($\boldsymbol{R^b}$), $\boldsymbol{\Phi}$ can be found

$$\nabla\boldsymbol{\Phi} = \boldsymbol{R^b} = \boldsymbol{R} - \boldsymbol{R^s} \quad (8)$$

The integration of total differential of each load function ,$e.\ g.\psi_x$,between two consecutive paths (paths 1 and 2) is as follows: