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Modeling of memory-dependent derivative in a fibre-reinforced plate

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ABSTRACT

Enlightened by the Caputo fractional derivative, the present study deals with a novel mathematical model of magneto-thermoelasticity to investigate the transient phenomena for a fibre-reinforced thick plate having a heat source in the context of three-phase-lag model of generalized thermoelasticity, which is defined in an integral form of a common derivative on a slipping interval by incorporating the memory-dependent heat transfer. The upper surface of the plate is free of traction having a prescribed surface temperature while the lower surface rests in a rigid foundation and is thermally insulated. Employing Laplace and Fourier transforms as tools, the problem has been solved analytically in the transformed domain. The inversion of the Fourier transform is carried out using suitable numerical techniques while the numerical inversion of Laplace transform is done incorporating a method on Fourier series expansion technique. According to the graphical representations corresponding to the numerical results, conclusions about the new theory is constructed. Excellent predictive capability is demonstrated due to the presence of memory dependent derivative, magnetic field and reinforcement also.

1. Introduction

In recent years, the theory of magneto-thermoelasticity which deals the interactions among the strain, temperature and magnetic field has drawn the attention of several researchers due to its extensive uses in diverse fields, such as geophysics, for understanding the effects of the earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices etc. [1].

Fibre-reinforced materials have extensive applications in aerospace and automotive fields, as well as in sailboats, and notably in modern bicycles and motorcycles, where their high strength-to-weight ratio is of great importance. Materials such as resins reinforced by strong aligned fibers exhibit highly anisotropic elastic behavior in the sense that their elastic moduli for extension in the fibre direction are frequently the order of 50 or more times greater than their elastic moduli in transverse extension or in shear. The mechanical behavior of many fibre-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction [2,3]. In such composites, the fibers are usually arranged in parallel straight lines. However, other configurations are used. An example is that of circumferential reinforcement, for which the fibers are arranged in concentric circles, giving strength and stiffness in the tangential (or hoop) direction.

The theory of generalized thermoelasticity has drawn attention of researchers due to its applications in various diverse fields such as engineering, nuclear reactor's design, high energy particle accelerators, etc. Actually, as is well known, the term ‘generalized’ usually refers to thermodynamic theories based on hyperbolic-type (wave-type) heat equations, so that a finite speed for propagation of thermal signal is admitted. Because of the experimental evidences in the support of finiteness of the heat propagation speed. Very recently, employing the generalized thermoelasticity theories, several remarkable studies have been reported [4–6]. One of these modern theories, the so-called three-phase-lag model, was proposed by Roychoudhuri [7]. According to this model

$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\dot{q}_i=-K\left(1+\tau_T\frac{\partial}{\partial t}\right)\dot{T}_{,i}-K^{\star}\left(1+\tau_\nu\frac{\partial}{\partial t}\right)T_{,i}, \tag{1}$$

where $T_{,i}$ is the temperature gradient at a point P of the material at time $t+\tau_T$, q_i is the component of heat flux at the point P in time $t+\tau_q$, K is the thermal conductivity of the material and $K^{\star}$ is additional material constant [8,9]. The delay time τ_T is interpreted as that caused by the microstructural interactions and is called the phase-lag of the temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of thermal inertia and is called the phase-lag of the heat flux and τ_ν is the phase-lag for thermal displacement gradient.

Diethelm [10] has developed the Caputo [11,12] derivative to be:

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$$D_0^{\zeta} f(t) = \int_0^t K_{\zeta}(t-\xi) f^{(m)}(\xi) d\xi, \tag{2}$$

where

$$K_{\zeta}(t-\xi) = \frac{(t-\xi)^{m-\zeta-1}}{\Gamma(m-\zeta)}, \tag{3}$$

and $f^{(m)}$ indicates the usual $m-$ th order derivative of the function. Differential equations of fractional order have been the focus of many studies due to their frequent appearance in various application in fluid mechanics, viscoelasticity, biology, physics, and engineering. The most important advantage of using fractional differential equations in these and other applications is their non-local property. It is well known that the integer order differential operator is a local operator but the fractional order differential operator is non-local [13].

From (2) and (3), it can be visualized that for any real number ζ, the kernel $K_{\zeta}(t-\xi)$ is a fixed function. But from the viewpoint of applications, different processes need different kernels to reflect their memory effects, so the kernel should be chosen freely. In fact, the memory effect of a real process basically occurs on a segment of time, i.e., on the delayed interval $[t-\omega, t]$ ($\omega(>0)$ indicates the time-delay). Enlightened by these, the novel concept of derivative was initiated as the "memory-dependent derivative" (MDD) to reflect the memory effect in a distinct manner. One may state that the definition of MDD is more intutionistic in realizing the physical significance and accordingly, the corresponding memory-dependent differential equations are more effective in real-world problems. Quite recently, introducing the concept of MDD, a few pioneering works can be reviewed from the following literatures [14–19].

Wang and Li [20] introduced a memory-dependent derivative, the first order of function f which is simply defined in an integral form of a common derivative with a kernel function on a slipping interval as follows

$$D_{\omega} f(t) = \frac{1}{\omega} \int_{t-\omega}^t K(t-\xi) f'(\xi) d\xi \tag{4}$$

where $\omega(>0)$ is the delay time and $K(t-\xi)$ is the kernel function in which they can be chosen freely, such as $K(t-\xi)=1$, $1-\frac{t-\xi}{\omega}$, $1-(t-\xi)$ and $(1-\frac{t-\xi}{\omega})^2$. The kernel function can be understood as the degree of the past effect on the present. In addition, if $K \equiv 1$,

$$D_{\omega} f(t) = \frac{1}{\omega} \int_{t-\omega}^t f'(\xi) d\xi = \frac{f(t)-f(t-\omega)}{\omega} \to f'(t). \tag{5}$$

So, the common derivative $\frac{d}{dt}$ can be seen as the limit of D_{ω} as $\omega \to 0$.

The right side of (4) can be understood as mean value of $f'(\xi)$ on the past interval $[t-\omega, t]$ with different weights. Generally, from the viewpoint of applications, the memory effect requires weight $0 \leqslant K(t-\xi) < 1$ for $\xi \in [t-\omega, t)$, so the magnitude of the memory dependent derivative is usually smaller than that of the common derivative $f'(t)$. The variational principles, reciprocal theorems and uniqueness of solutions due to memory dependence in a thermodiffusive medium have been proved by El-Karamany and Ezzat [21].

Further, following the definition (4), the constitutive law for the heat flux under memory-dependent 3P lag model can be represented as

$$\left(1+\frac{\tau_q}{1!}D_{\omega_1}+\frac{\tau_q^2}{2!}D_{\omega_1}^2\right)\dot{q}_i = -K\left(1+\frac{\tau_T}{1!}D_{\omega_2}\right)\dot{T}_{,i} - K^{\star}\left(1+\frac{\tau_\nu}{1!}D_{\omega_3}\right)T_{,i}, \tag{6}$$

where ω_1, ω_2 and ω_3 are the delay times due to 3P lag model.

The main objective of the paper is to study a two dimensional problem of a magneto-thermoelastic fibre-reinforced thick plate in the context of memory dependent generalized thermoelastic 3P lag model in presence of heat source, from which GN III model can be obtained as a particular case of the problem. The upper surface of the plate is stress free with prescribed surface temperature while the lower surface of the plate is laid down on a rigid foundation and is thermally insulated. The governing equations are solved in Laplace-Fourier double transform domain by applying Laplace and Fourier transform techniques. The inversions of double transform has been done numerically. The numerical inversion of Laplace transform is done by using a method based on Fourier series expansion technique [22]. Numerical computation have been done for Magnesium (Mg) and the results are presented graphically for different theories (three-phase-lag model and GN III model). Excellent predictive capability is demonstrated due to the influence of the magnetic field, the presence of reinforcement and memory-dependent derivative also.

2. Formulation of the problem

We consider an infinite fibre-reinforced thermally conducting thick plate with spatially varying heat source at an uniform reference temperature T_0 in the undisturbed state. The adjacent space is assumed to be permeated by a uniform magnetic field $\mathbf{H}$ acting perpendicular to the boundary $z=0$. This produces an induced magnetic field $\mathbf{h}$ and induced electric field $\mathbf{E}$ which satisfies the linearized equations of electro-magnetism and are valid for slowly moving media. The electromagnetic field is governed by the Maxwell's equation as follows

$$\begin{aligned} &\text{curl } \mathbf{h} = \mathbf{J} + \varepsilon_0 \dot{\mathbf{E}}, \\ &\text{curl } \mathbf{E} = -\mu_0 \dot{\mathbf{h}}, \\ &\mathbf{E} = -\mu_0(\dot{\mathbf{u}} \times \mathbf{H}), \\ &\text{div } \mathbf{h} = 0. \end{aligned} \tag{7}$$

We set $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$, where $\mathbf{H}_0 = (0, 0, H_0)$. The perturbed magnetic field $\mathbf{h}$ is so small that the product of $\mathbf{h}$ and $\mathbf{u}$ and their derivatives can be neglected for linearization of field equations. Here, $\mathbf{J}$ is the electric current density vector, $\mathbf{u}$ is the displacement vector. μ_0 and ε_0 are the magnetic permeability in vacuum and electric permeability in vacuum, respectively.

The upper surface of this medium is taken traction free and subjected to a known temperature distribution. The lower surface of the plate is laid down on a rigid foundation and is thermally insulated. Let the faces of the plate be the planes $x = \pm h$, referred to a rectangular set of cartesian co-ordinates axes Ox, Oy and Oz as shown in Fig. 1. We shall consider two dimensional deformation of the plate parallel to xy plane.

The displacement vector $\mathbf{u}$ and temperature T can be taken in the following form

$$\begin{aligned} &\mathbf{u} = (u(x,y,t), v(x,y,t), 0), \\ &T = T(x,y,t). \end{aligned} \tag{8}$$

The constitutive equation for a fibre-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit

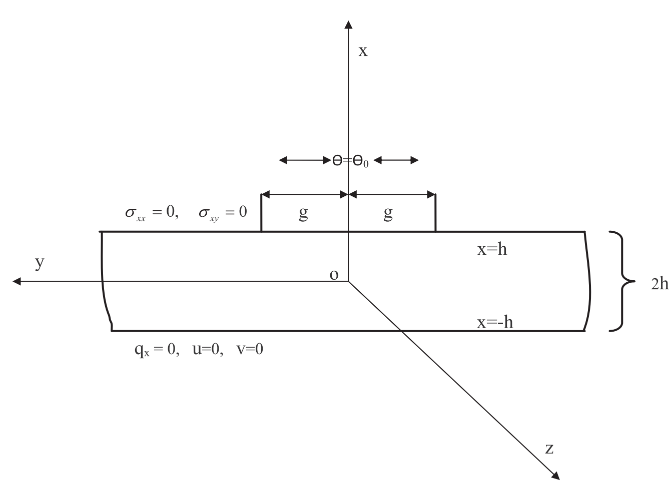

Fig. 1. Co-ordinate system and geometry of the plate with boundary conditions.