Thin-Walled Structures 127 (2018) 710–727

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Smart damping of large amplitude vibrations of variable thickness laminated composite shells

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ABSTRACT

This paper is concerned with the smart constrained layer damping (SCLD) treatment of laminated composite shells with variable thickness undergoing geometrically nonlinear vibrations. Three dimensional fractional derivative model (FDM) has been implemented for modelling the constrained viscoelastic layer of the SCLD treatment. The constraining layer of the SCLD treatment is made of vertically/obliquely reinforced 1–3 piezoelectric composites (PZCs) and acts as the distributed actuator. The strain-displacement relations are based on the simplified Novozhilov nonlinear shell theory to introduce the geometric nonlinearity in the large amplitude vibrations of the variable thickness shells. A three dimensional smart nonlinear finite element (FE) model has been developed for carrying out this analysis. Several numerical results are presented to check the accuracy of the present three-dimensional FDM for analyzing the passive and active control authority of the SCLD patch. Also the efficacy of the activated SCLD patch in controlling geometrically nonlinear vibration is computed for variable thickness shells and compared with shells of constant thickness.

1. Introduction

The demand for the use of lightweight composite shell structures has been continuously increasing in aerospace, automotive and marine industries. In the past researchers have theoretically and experimentally studied the linear and geometrically nonlinear vibrations of thin and moderately thick composite shell structures [1–8] with constant shell thickness along both the principal directions of the domain. More recently plate and shell structures with variable thickness are gaining lot of attention as they may exhibit better performance under various loading conditions compared to shells with constant thickness [9]. Further, a variation in thickness offers variation in stiffness and the shape of the structure can be optimized while the weight remains unaltered. Back in 1970, Lord and Yousef [10] started the analytical and experimental study on the effect of thickness variation in case of isotropic annular and circular thin plates. They used a simple FE model considering the plate to be composed of several concentric rings with constant thickness. Later, several researchers have shown interest to study the static and dynamic behavior of variable thickness plates and shells [11–26]. Sherbourne and Murthy [11] analytically computed the symmetrical bending of orthotropic circular plates with variable thickness. Sinharay and Banerjee [12] investigated large-amplitude free vibrations of thin elastic shallow spherical and cylindrical shells with variable thickness for various edge conditions. An energy method based on the Rayleigh-Ritz procedure has been used by Sankaranarayanan et al. [13] to perform free vibration analysis of laminated conical shells with a linear variation of thickness in the meridional direction. Sivadas and Ganesan [14] employed a semi-analytical FE method to determine the natural frequencies of thin cylindrical shells with linear and quadratic thickness variation along the axial direction. Suzuki et al. [15] presented an analytical solution procedure to analyse free vibrations of rotating circular cylindrical shells with variable thickness in the axial direction. A comprehensive study on the modelling of vibration analysis of variable thickness cantilevered shallow cylindrical shells of rectangular planform is carried out by Liew and Lim [16]. Leissa and Kang [17] performed free vibration analysis of moderately thick and thick paraboloidal shells with variable thickness. They obtained numerical results for a variety of shallow and deep shells having uniform or

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https://doi.org/10.1016/j.tws.2018.03.009
Received 13 October 2017; Received in revised form 6 February 2018; Accepted 6 March 2018
0263-8231/

Nomenclature

A_j Grünwald constant
$[\overline{C}_b^k]$, $[\overline{C}_s^k]$ Transformed elastic coefficient matrices of the substrate shell with respect to the laminate coordinate system (GPa)
$[C_b^{N+1}]$, $[\overline{C}_b^{N+1}]$, $[C_s^{N+1}]$, $[\overline{C}_s^{N+1}]$ Elastic coefficient matrices of the viscoelastic material (MPa)
$[C_b^{N+2}]$, $[C_s^{N+2}]$, $[C_{bs}^{N+2}]$ Elastic coefficient matrices and elastic coupling matrix of the 1–3 piezocomposite material (GPa)
$[C_n]$ Global active damping matrix (Ns/m)
D_z Electric displacement along the z-direction (C/m^2)
$\{d_t\}$, $\{d_r\}$ Generalized translational and rotational vectors, respectively
$\{d_t^e\}$, $\{d_r^e\}$ Nodal generalized translational and rotational vectors, respectively
E_0, E_∞ Relaxed and non-relaxed elastic moduli of the viscoelastic material (MPa)
E_1, E_2, E_3 Modulus of elasticity of the composite substrate shell (Gpa)
E_z Applied electric field component in the z direction (C/m or V/m)
$\{e_b\}$, $\{e_s\}$ Piezoelectric coefficient matrices (C/m^2)
$\{F^e\}$ Elemental load vector (N)
$\{F\}_{n+1}$ Global nodal force vector at $(n + 1)$th time step (N)
$\{\overline{F}_{tn}^e\}$, $\{\overline{F}_{rn}^e\}$ Elemental memory load vectors due to the viscoelastic material (N)
$\{\overline{F}_{tn}\}_{n+1}$, $\{\overline{F}_{rn}\}_{n+1}$ Global viscoelastic memory load at $(n + 1)$th time step (N)
$\{\overline{F}_n\}_{n+1}$ Global nonlinear viscoelastic load vector at $(n + 1)$th time step (N)
$\{F_{tpn}^e\}$, $\{F_{rp}^e\}$ Elemental electro-elastic coupling vectors (C/m)
$\{F_{tpn}\}$, $\{F_{rp}\}$ Global electro-elastic coupling vectors (C/m)
G_0, G_∞ Relaxed and non-relaxed shear moduli of the viscoelastic material (MPa)
G_{12}, G_{13}, G_{23} Shear modulus of the composite substrate shell (Gpa)
$h(\alpha_x, \alpha_y)$, h_v, h_p Variable thickness of the shell, constant thickness of the constrained viscoelastic layer and the constraining 1–3 piezoelectric composite layer, respectively (m)
h_0 Reference thickness of the variable thickness shell (m)
h_{k+1}, h_k Thickness co-ordinates z of the top and the bottom surfaces of the kth layer (m)
I_d Performance index
k_c Shear correction factor
K_d Control gain
$[K_{ttn}^e]$, $[K_{trn}^e]$, $[K_{rtn}^e]$ Elemental nonlinear stiffness matrices (N/m)
$[K_{ttn}]$, $[K_{trn}]$, $[K_{rtn}]$ Global nonlinear stiffness matrices (N/m)
$[K_{rr}^e]$ Elemental linear stiffness matrix (N/m)
$[K_{rr}]$ Global linear stiffness matrix (N/m)
$[K_n]$ Global nonlinear stiffness matrix (N/m)
L_x, L_y Curvilinear lengths of the substrate shell along α_x and α_y axis, respectively (m)
$[M^e]$, $[M_r^e]$ elemental mass matrices (kg)
$[M]$, $[M_r]$ Global mass matrices (kg)
N Number of layers in substrate shell
N_t Number of terms in Grünwald series
$[N_t^s]$, $[N_r^s]$ Shape function matrices
R_1, R_2 Principal radii of curvature of the middle surface of the shell (m)
u, v, w Displacements along the α_x, α_y and z directions, respectively (m)
u_0, v_0, w_0 Displacements of a point on the reference mid-plane along the α_x, α_y and z directions, respectively (m)
V_{n+1} Applied voltage across the thickness of the piezoelectric layer (Volt)
$\{X\}_{n+1}$, $\{X_r\}_{n+1}$ Global nodal generalized displacement vectors
$\alpha_x\alpha_y z$ Generic laminate co-ordinate system
α Fractional order of the time derivative ($0 < \alpha < 1$)
$\hat{\alpha}_r$, $\hat{\varsigma}_r$, $\hat{\omega}_r$ Three positive constant parameters of the GHM model
β_x, β_y Generalized rotations of the normal to the middle plane of the viscoelastic layer (rad)
β_z Second order derivative of the transverse displacement in the overall structure with respect to the thickness co-ordinate (rad/m)
γ_{xy}^k In-plane shear strain at any point in the kth layer
γ_{xz}^k, γ_{yz}^k Transverse shear strains at any point in the kth layer
ε_x^k, ε_y^k, ε_z^k Normal strains along the α_x, α_y and z directions in the kth layer, respectively
$\{\varepsilon_b^k\}$, $\{\varepsilon_s^k\}$ Bending and transverse strain vectors
ε_x^{ul}, ε_y^{ul}, ε_z^{ul} Uniaxial normal strains along the α_x, α_y and z directions, respectively under uniaxial loading alone
$\{\bar{\varepsilon}_b^{N+1}\}$, $\{\bar{\varepsilon}_s^{N+1}\}$ Anelastic bending and transverse strain vectors
$\in_{33}$ Dielectric constant of the piezoelectric material
λ_x, λ_y Generalized rotations of the normal to the middle plane of the PZC layer (rad)
θ_k Fiber orientation angle in the kth layer of the shell with respect to the α_x-axis (rad)
θ_x, θ_y, θ_z Generalized rotations of the normal to the middle plane of the substrate shell (rad)
ρ^k Mass density of the kth layer (kg/m^3)
σ_x^k, σ_y^k, σ_z^k Normal stresses along the α_x, α_y and z directions in the kth layer, respectively
σ_{xy}^k In-plane shear stress at any point in the kth layer
σ_{xz}^k, σ_{yz}^k Transverse shear stresses at any point in the kth layer
τ Relaxation time of the viscoelastic material (s)
ψ Piezoelectric fiber orientation in the vertical plane of the PZC layer with respect to the z-axis (rad)
ω_l Linear natural frequency of the overall structure (rad/s or Hz)
ω_{nl} Nonlinear natural frequency of the overall structure (rad/s or Hz)

variable thickness. In another work, Kang and Leissa [18] presented a three-dimensional method of analysis of solid paraboloids and paraboloidal shells of revolution with variable thickness. Zenkour [19] analytically investigated the deformation and stresses in composite circular cylinders of axially variable thickness. A numerical-analytic approach was proposed by Budak et al. [20] to study the free vibrations of orthotropic shallow shells. They also reported the influence of the mid-surface curvature and variable thickness on the behavior of dynamic characteristics of the structures. Duan and Koh [21] analytically obtained the benchmark solutions for the cylindrical shells with thickness varying monotonically in arbitrary power form. Kurpa and Chistilina [22] studied natural vibrations of orthotropic shells with varying thickness using the R-function and Ritz methods. Efraim and Eisenberger [23] numerically determined natural frequencies and mode shapes of thick spherical shell segments with linearly varying thickness and different boundary conditions based on dynamic stiffness method. Amabili [24] presented a new nonlinear higher-order shear deformation theory for large-amplitude vibrations of laminated doubly curved