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GBT-based cross-section deformation modes for curved thin-walled members with circular axis

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ABSTRACT

This paper presents an improvement of the first-order Generalised Beam Theory (GBT) formulation proposed in [1], which was developed for naturally curved thin-walled members with deformable cross-section and whose undeformed axis is a circular arc with no pre-twist. In this paper, the restrictions on the cross-section shape are removed (in the previous paper only rather simple cross-sections were dealt with) by proposing and discussing a novel and systematic procedure to obtain the cross-section deformation modes for arbitrary flat-walled cross-sections (open, closed or "mixed"). The proposed procedure retains the nomenclature of the deformation mode subsets defined in [2,3], even though the kinematic constraints employed to subdivide the modes are much more complex than for prismatic members. A set of representative illustrative examples is presented, involving complex local-distortional-global deformation, to show the efficiency of the proposed procedure when used together with a standard displacement-based GBT finite element. It is demonstrated that extremely accurate results are obtained with rather few DOFs and that the GBT modal solution provides in-depth insight into the structural behaviour of naturally curved members.

1. Introduction

Generalised Beam Theory (GBT) is a thin-walled bar theory that incorporates cross-section in-plane and out-of-plane (warping) deformation through the addition of hierarchical and structurally meaningful cross-section DOFs, the so-called "cross-section deformation modes". GBT was proposed and initially developed by Schardt and co-workers [4,5] [1], being presently well-established as an efficient, versatile, accurate and insightful approach to assess the structural behaviour of prismatic thin-walled bars (see, e.g., [6–8] [2]).

Allowance for cross-section deformation in curved thin-walled bars is a subject with significant practical interest. The so-called classical formulations for bridges employ a single distortional deformation mode [9,10] and are solved using a beam on elastic foundation analogy. For closed box girders, this distortional deformation mode was recently shown to be identical to the so-called Vlasov distortional mode obtained with GBT [11]. Nevertheless, examples of this type of approach are still being employed (see e.g. [12] and references therein). In [13], a formulation is presented for curved beams that includes warping modes to account for shear deformation due to bending and torsion. However, cross-section in-plane deformation is not considered.

Quite recently, in [1], the authors have proposed, for the first time, a linear GBT formulation for elastic thin-walled bars with circular axis (without pre-twist). This formulation extends the classic prismatic case — thus can handle virtually arbitrary cross-section deformation — while still making it possible to incorporate the usual GBT strain assumptions: (i) Kirchhoff's (thin plate), (ii) Vlasov's (null membrane shear strains) and (iii) null membrane transverse extensions. The equilibrium equations were derived in terms of both GBT modal matrices and stress resultants and it was demonstrated that, for the so-called "rigid-body" deformation modes (extension, bending and torsion), the equations coincide with those of the Winkler (in-plane case [14]) and Vlasov (out-of-plane case [15]) classical theories. A standard displacement-based GBT finite element was employed to show that complex local-global deformation can be efficiently and accurately captured with the proposed formulation.

The formulation presented in [1] can handle all types of deformation modes, but their systematic determination for complex cross sections was postponed to the present paper. This was due to the fact that the so-called "natural Vlasov modes" (those complying with Vlasov's

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[1] See http://www.gbt.info for a complete list of publications by this group.
[2] A list of publications by the Lisbon-based research group is available at http://www.civil.ist.utl.pt/gbt.

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assumption) need to be calculated using a constraint that is significantly more complex for curved bars. For this reason, only cross-sections without Vlasov *distortional* modes were addressed. To obtain the rigid-body modes (extension, bending and torsion) for open sections, a two-step procedure was employed, where (i) the warping functions for prismatic bars were first calculated with the GBTUL program[3] and, subsequently, (ii) the cross-section in-plane displacements were calculated from the warping functions, using Vlasov's assumption for curved bars. For closed sections, the extension and bending modes were obtained in the same way, but the torsion and shear distortional modes for prismatic bars were directly employed.

The present paper completes the previous work by establishing a systematic procedure for the determination of all cross-section deformation modes for members with circular axis, extending the concepts introduced for the prismatic case in [2,3,16]. In particular, the deformation mode categorisation employed in these papers is preserved and all types of flat-walled cross-sections are considered, namely open, closed or “mixed” (with open and closed parts). The fact that the deformation modes are hierarchical and subdivided according to specific kinematic constraints renders the GBT analyses quite efficient, (i) leading to accurate solutions with only a few deformation modes and much less DOFs than those typically required in shell finite element models, and (ii) providing in-depth insight into the mechanics of the problem under consideration, through the modal decomposition of the solution.

The outline of the paper is as follows. First, Section 2 reviews the fundamental equations of the GBT formulation for naturally curved beams, as proposed in [1]. Section 3 presents the proposed procedure for calculating the cross-section deformation modes. The Vlasov and null membrane transverse extension assumptions are introduced and employed to divide the modes, as this contributes decisively to the overall efficiency of the formulation, by reducing significantly the number of modes required to achieve accurate solutions. Next, in Section 4, a set of representative numerical examples, involving complex local-distortional-global deformation patterns, is presented and solved using a standard displacement-based GBT finite element. For comparison purposes, solutions obtained with refined shell finite element models are also given. The paper closes in Section 5, with the concluding remarks.

The notation follows closely that introduced in [2,17,18]. Moreover, the subscript commas indicate derivatives (e.g., $f_{,x} = \partial f/\partial x$), although the prime is reserved for a derivative with respect to the beam axis arc-length X, i.e. $(\cdot)' = \partial(\cdot)/\partial X$. Finally, superscripts $(\cdot)^M$ and $(\cdot)^B$ designate plate-like membrane and bending terms, respectively.

2. First-order GBT for members with circular axis

In this section, the formulation presented in [1] is briefly reviewed, for completeness of the paper. Fig. 1 shows a naturally curved thin-walled member with flat-walled cross-section (left) and a 2D view of a single wall (right). The global cylindrical coordinate system is (θ, Z, R), X is the member axis arc-length coordinate, lying on the $Z = Z_C$ plane and having constant curvature equal to $1/R_C$, where C is an arbitrary cross-section “centre” (the intersection of the member axis with each cross-section). Finally, the wall local axes are defined by (x, y, z), where y and z stand for the wall mid-line and through-thickness directions, respectively, and x is concentric to X.

The usual GBT variable separation technique for the membrane displacement components makes it possible to write

$$u^M = \sum_{k=1}^{D} \bar{u}_k(y)\, \phi'_k(X) = \bar{\boldsymbol{u}}^T(y)\, \boldsymbol{\phi}'(X), \tag{1}$$

$$v^M = \sum_{k=1}^{D} \bar{v}_k(y)\, \phi_k(X) = \bar{\boldsymbol{v}}^T(y)\, \boldsymbol{\phi}(X), \tag{2}$$

$$w^M = \sum_{k=1}^{D} \bar{w}_k(y)\, \phi_k(X) = \bar{\boldsymbol{w}}^T(y)\, \boldsymbol{\phi}(X), \tag{3}$$

where $\bar{u}_k, \bar{v}_k, \bar{w}_k$ are the mid-line displacement components of deformation mode k along the local axes (x, y, z), respectively, D is the number of modes, ϕ_k are the corresponding amplitude functions and $\bar{\boldsymbol{u}}, \bar{\boldsymbol{v}}, \bar{\boldsymbol{w}}, \boldsymbol{\phi}$ are column vectors collecting $\bar{u}_k, \bar{v}_k, \bar{w}_k, \phi_k$, respectively. As shown in [1], using Kirchhoff's thin plate assumption, the displacement field in the local axes can be expressed as

$$\boldsymbol{U} = \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = \Xi_U \begin{bmatrix} \boldsymbol{\phi} \\ \boldsymbol{\phi}' \end{bmatrix}, \tag{4}$$

with the auxiliary matrix

$$\Xi_U = \begin{bmatrix} \mathbf{0} & \bar{\boldsymbol{u}}^T + zK_y\bar{\beta}\bar{\boldsymbol{u}}^T - z\bar{\beta}\bar{\boldsymbol{w}}^T \\ \bar{\boldsymbol{v}}^T - z\bar{\boldsymbol{w}}_{,y}^T & \mathbf{0} \\ \bar{\boldsymbol{w}}^T & \mathbf{0} \end{bmatrix}, \tag{5}$$

where K_y and K_z are the beam axis curvatures along the wall local axes, i.e.,

$$K_y = \frac{\cos\varphi}{R_C}, \quad K_z = -\frac{\sin\varphi}{R_C}, \tag{6}$$

and a mid-line parameter was introduced,

$$\bar{\beta} = \frac{R_C}{\bar{R}}, \tag{7}$$

where $\bar{R}$ is the mid-line radius (see Fig. 1, which shows φ, R and $\bar{R}$ for an arbitrary point P).

The strains are subdivided into membrane and bending components, being given by

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^M + \boldsymbol{\varepsilon}^B = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \Xi_\varepsilon \begin{bmatrix} \boldsymbol{\phi} \\ \boldsymbol{\phi}' \\ \boldsymbol{\phi}'' \end{bmatrix}, \tag{8}$$

$$\boldsymbol{\varepsilon}^M = \Xi_\varepsilon^M \begin{bmatrix} \boldsymbol{\phi} \\ \boldsymbol{\phi}' \\ \boldsymbol{\phi}'' \end{bmatrix}, \quad \boldsymbol{\varepsilon}^B = \Xi_\varepsilon^B \begin{bmatrix} \boldsymbol{\phi} \\ \boldsymbol{\phi}' \\ \boldsymbol{\phi}'' \end{bmatrix}, \tag{9}$$

$$\Xi_\varepsilon = \Xi_\varepsilon^M + \Xi_\varepsilon^B, \tag{10}$$

$$\Xi_\varepsilon^{(\cdot)} = \begin{bmatrix} (\boldsymbol{\xi}_{11}^{(\cdot)})^T & \mathbf{0} & (\boldsymbol{\xi}_{13}^{(\cdot)})^T \\ (\boldsymbol{\xi}_{21}^{(\cdot)})^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\boldsymbol{\xi}_{32}^{(\cdot)})^T & \mathbf{0} \end{bmatrix}, \tag{11}$$

where $\boldsymbol{\xi}_{ij}^{(\cdot)}$ are column vectors reading

$$\boldsymbol{\xi}_{11}^M = \bar{\beta}\,(K_y\bar{\boldsymbol{w}} - K_z\bar{\boldsymbol{v}}), \tag{12}$$

$$\boldsymbol{\xi}_{13}^M = \bar{\beta}\bar{\boldsymbol{u}}, \tag{13}$$

$$\boldsymbol{\xi}_{21}^M = \bar{\boldsymbol{v}}_{,y}, \tag{14}$$

$$\boldsymbol{\xi}_{32}^M = \bar{\beta}\bar{\boldsymbol{v}} + \bar{\beta}K_z\bar{\boldsymbol{u}} + \bar{\boldsymbol{u}}_{,y}, \tag{15}$$

$$\boldsymbol{\xi}_{11}^B = -z\bar{\beta}\,(-K_z\bar{\boldsymbol{w}}_{,y} + \bar{\beta}K_y^2\bar{\boldsymbol{w}} - \bar{\beta}K_yK_z\bar{\boldsymbol{v}}), \tag{16}$$

$$\boldsymbol{\xi}_{13}^B = -z\bar{\beta}^2\bar{\boldsymbol{w}}, \tag{17}$$

$$\boldsymbol{\xi}_{21}^B = -z\bar{\boldsymbol{w}}_{,yy}, \tag{18}$$

$$\boldsymbol{\xi}_{32}^B = -z\bar{\beta}\,(2\bar{\boldsymbol{w}}_{,y} + 2\bar{\beta}K_z\bar{\boldsymbol{w}} - K_y\bar{\boldsymbol{u}}_{,y} + \bar{\beta}K_y\bar{\boldsymbol{v}} - \bar{\beta}K_yK_z\bar{\boldsymbol{u}}). \tag{19}$$

[3] This program is freely available at http://www.civil.ist.utl.pt/gbt.