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Buckling analysis of thin-walled members with open-branched cross section via semi-analytical finite strip transfer matrix method

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A B S T R A C T

Thin-walled members with open-branched cross section have been used in many modern engineering structures, and their buckling performance has been widely studied. In this paper, two transfer methods are developed to tackle the transfer problems at the junction of the open-branched cross-section of thin-walled members by using semi-analytical finite strip transfer matrix method (FS-TMM) which is a combined use of semi-analytical finite strip method (SA-FSM) and transfer matrix method (TMM) for the buckling analysis. Compared to traditional SA-FSM, this method has a smaller matrix and higher computational efficiency due to no global stiffness matrix generated. An asymmetric E-section member, a symmetric I-section member and a X-section member with loaded edges simply supported are analyzed by the derived formulation. All the results are compared with SA-FSM's or finite element method's results to prove the reliability and efficiency of this method.

1. Introduction

In order to lighten engineering structures and reduce the cost of production, thin-walled structures are widely used in mechanical construction, civil architecture, aerospace and other engineering fields [1]. Due to the rapid increase of requirement, many engineers and scientists give priority to the study of thin-walled structures with complex cross-sections and longitudinal shapes [2].

During the design process of slender thin-walled structures, the buckling possibility is the most important factor to determine the parameters and configurations of the structures [3]. The buckling and vibration of any long, thin and flat structures, subjected to a basic state of plane stress, were analyzed by the derived stiffness matrices in 1968 [4], after which the finite element displacement method was used to analyze the post-buckling behavior and the ultimate strength of thin-walled non-planar structural members by considering both geometrical nonlinearity and material non-linearity [5]. Up to now, many methods have been used to tackle the buckling problems of thin-walled structures, including finite difference method [6], finite layer method [7], generalized beam theory (GBT) [8,9], semi-analytical finite strip method (SA-FSM), spline finite strip method [10], constrained finite strip method (cFSM) [11], and based on GBT and cFSM, a finite element procedure was employed to the linear buckling analysis of thin-walled structures [12]. Besides, the direct strength method is a method used for cold-formed steel member design [13].

For the sake of reducing the element number and improving the computational efficiency of the system, finite strip method (FSM) was developed to analyze the prismatic structures [14]. If the sub-parametric mapping concept is adopted, the FSM can be used to discretize structures which only have complex geometry in their cross-sections [15]. The buckling stresses and natural frequencies of prismatic plate structures can be predicted by introducing the spline finite strip method [16], which is then combined with modified couple stress theory for stability analysis of thin FGM microplate subjected to mechanical and thermal loading [17]. The SA-FSM, which is innovated from the concept of the semi-energy approach, can be used to analyze the buckling [18], shear buckling [19], dynamic buckling of plate structures [20]. It has also been implemented in open source programs (such as THIN-WALL [21], CUFSM [22]) to generate the signature curves [23] of the buckling coefficient versus buckling half-length for thin-walled members. In addition, the cFSM extended from SAFSM's solutions has been developed and applied to determinate and classify buckling modes [24]. In order to study prismatic members with arbitrary cross-sections, the cFSM has been extended to buckling modes decomposition for prismatic members with branches [25].

In addition to FEM, the classical transfer matrix method (TMM) is also used widely in structure analysis, especially for chain connected system from topological perspective [26]. The combination of the finite element and transfer matrix methods has generated a new method, named as finite element-transfer matrix method (FE-TMM), to analyze the static and dynamic structure problems [27]. By combining the boundary element method and TMM, a structural analysis method

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called boundary element-transfer method(BE-TMM) is proposed for continuous plate bending problems [28]. If a numerical integration can be introduced, the TMM can be used to study the dynamics of multi-rigid-body system, multi-flexible-body system [29] and multi-rigid-flexible-body system [30]. For stability problems, the strip transfer function method is developed to investigate the buckling of plate with built-in rectangular delamination [31], the TMM can be used to analyze the instability in unsymmetrical rotor-bearing systems [32] and tall unbraced frames [33]. TMM has two major characteristics generally: 1 TMM formulae share similarity to the chain mechanics model in terms of topology structure; 2 TMM is adopted to deal with the problems of the discrete system, continuous system, and especial discrete/continuous coupling system in the same form. The topology comparability between the mechanics model and its corresponding formulae of TMM can be adopted to assembling the transfer matrices and transfer equations of the global tree system [34]. An absorbing transfer matrix method in frequency domain for fluid-filled pipelines with any branched pipes has been proposed [35]. The buckling analysis of rectangular thin plates and single-branched thin-walled members via FS-TMM has been developed recently [36,37].

We also noted that the thin-walled members with arbitrarily branched open cross-sections can be analyzed by GBT [38]. It enlightens us to start this investigation. Here FS-TMM can be extended to analyze the buckling problems of thin-walled member with open, branched thin-walled cross-sections. The paper is organized as follows: In Section 2, the general theorem of the semi-analytical finite strip for buckling analysis of thin-walled member is shown. In Section 3, the FS-TMM for sections with branches is studied using two transfer strategies. Section 4 gives three examples calculated by proposed two strategies, and the results are compared with FSM or FEM results to validate the method. The conclusions are presented in Section 5.

2. The semi-analytical finite strip analysis

2.1. Degree of freedom and shape function

The SA-FSM can be considered as a special form of the FEM, in which a prismatic member can be discretized into many strips in longitudinal direction. Fig. 1a shows a T-section member, which is the structure with the simplest open-branched cross section. For a single strip, the nodal line $i(j)$ has two membrane degrees of freedom (DOFs) $u_{i(j)}$ and $v_{i(j)}$, two bending DOFs $w_{i(j)}$ and $\theta_{i(j)}$, as shown in Fig. 1b. The total number of strip is s, therefore, the total number of nodal line is $s+1$ in the open cross section with one branched point.

For the strip element with simply supported boundary condition at the loaded edges of the strip, the analytical trigonometric functions of the longitudinal coordinate can be used to represent the longitudinal deflected shape,

$$Y_p(y)=sin\frac{p\pi y}{a},\quad p=1,2,3,\cdots,m, \tag{1}$$

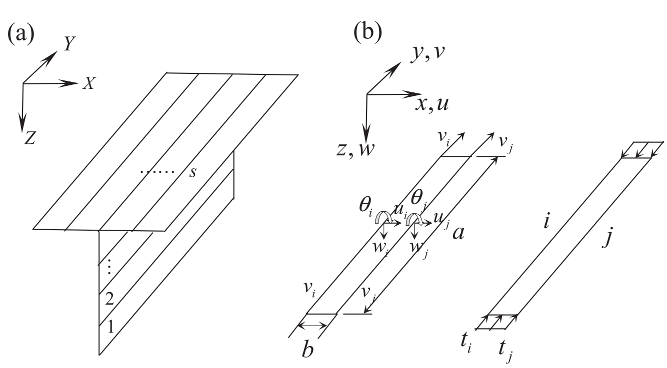

Fig. 1. Coordinate systems, displacements.

where p is the half-wave number, y is the longitudinal coordinate in local coordinate system, m is the maximum half-wave number as well as a finite positive integer employed in the analysis, a is the length of the member.

Both of the membrane DOFs and the bending DOFs of the strip should be considered during the buckling analysis of thin-walled structures. A linear function is employed as the shape function for the membrane DOFs along transverse direction x. The shape function for the bending DOFs can be selected as four cubic polynomials in the x direction. Then the explicit expressions of u, v and w can be given as follows,

$$\begin{Bmatrix} u \\ v \end{Bmatrix}=\sum_{p=1}^{m}\begin{bmatrix} Y_p & 0 \\ 0 & Y'_p\frac{a}{p\pi}\end{bmatrix}\begin{bmatrix} (1-\frac{x}{b}) & 0 & \frac{x}{b} & 0 \\ 0 & (1-\frac{x}{b}) & 0 & \frac{x}{b}\end{bmatrix}\begin{Bmatrix} u_{ip} \\ v_{ip} \\ u_{jp} \\ v_{jp}\end{Bmatrix}=\sum_{p=1}^{m}\mathbf{N}_{uv}\mathbf{D}_{uv}^{p}, \tag{2}$$

$$w=\sum_{p=1}^{m}[1-\frac{3x^2}{b^2}+\frac{2x^2}{b^3}x-\frac{2x^2}{b}+\frac{x^3}{b^2}\frac{3x^2}{b^2}-\frac{2x^3}{b^3}\frac{x^3}{b^2}-\frac{x^2}{b}]Y_p\begin{Bmatrix} w_{ip} \\ \theta_{ip} \\ w_{jp} \\ \theta_{jp}\end{Bmatrix}=\sum_{p=1}^{m}\mathbf{N}_{w}\mathbf{D}_{w}^{p}, \tag{3}$$

where subscripts i and j may express start nodal line and end nodal line of each strip element, m is the maximum half-wave number, $\mathbf{N}_{uv}$ and $\mathbf{N}_{w}$ are shape functions of membrane and bending respectively, $\mathbf{D}_{uv}^{p}$ and $\mathbf{D}_{w}^{p}$ are displacement vectors of the membrane DOFs and the bending DOFs of the strip correspondingly [37].

2.2. Fundamental stiffness matrix

The derivation process of the elastic stiffness matrix in the SA-FSM is similar to that of in the FEM. The total strain $\boldsymbol{\varepsilon}$, which is consisted by the membrane strains $\boldsymbol{\varepsilon}_M$ and the bending strains $\boldsymbol{\varepsilon}_B$, can be expressed as follows by using the plane stress assumptions and Kirchhoff plate theory respectively,

$$\boldsymbol{\varepsilon}_M=\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix}_M=\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix}_M=\sum_{p=1}^{m}\mathbf{B}_{M}^{p}\mathbf{D}_{uv}^{p}, \tag{4}$$

$$\boldsymbol{\varepsilon}_B=\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix}_B=\begin{Bmatrix} -z\frac{\partial^2 w}{\partial x^2} \\ -z\frac{\partial^2 w}{\partial y^2} \\ 2z\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix}_B=\sum_{p=1}^{m}z\mathbf{B}_{B}^{p}\mathbf{D}_{w}^{p}, \tag{5}$$

where $\mathbf{B}_{M}^{p}$ and $\mathbf{B}_{B}^{p}$ are the deformation matrixes, $\mathbf{D}_{uv}^{p}$ and $\mathbf{D}_{w}^{p}$ are the nodal displacements, the subscripts M and B denote the membrane and the bending effect correspondingly [22].

For general linear elastic material, the internal strain energy during buckling can be expressed as,

$$U=\frac{1}{2}\int_V\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}dV=\frac{1}{2}\int_V\boldsymbol{\varepsilon}^T\mathbf{E}\boldsymbol{\varepsilon}dV=\frac{1}{2}\mathbf{D}^T\left(\int_V\mathbf{B}^T\mathbf{E}\mathbf{B}dV\right)\mathbf{D}=\frac{1}{2}\sum_{p=1}^{m}\sum_{q=1}^{m}\mathbf{D}^{pT}\mathbf{k}_e^{pq}\mathbf{D}^{q}, \tag{6}$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ denote the strain vector and the stress vector respectively, V is the volume of the material, $\mathbf{B}$ is the deformation matrix containing the membrane term and the bending term in Eqs. (4) and (5), $\mathbf{D}$ is the displacement vector, $\mathbf{E}$ denotes the elastic constant matrix of the material [24].