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GBT-based assessment of the mechanics of distortional-global interaction in thin-walled lipped channel beams

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ABSTRACT

This work aims at presenting and discussing Generalised Beam Theory (GBT) numerical results concerning the elastic geometrically non-linear behaviour of simply supported lipped channel (LC) beams under uniform major-axis bending and experiencing distortional-global (D-G) interaction, making it possible to shed fresh light on the mechanics underlying this coupling phenomenon. Two LC beam geometries are considered, each exhibiting a different type of D-G interaction, namely (i) "true D-G interaction", associated with close distortional (M_{crD}) and global (M_{crG}) critical buckling moments, and (ii) "secondary global-bifurcation interaction – SGI", corresponding to $M_{crD} << M_{crG}$. While the latter beam geometry contains a critical-mode distortional initial imperfection, the former one is analysed with initial geometrical imperfections exhibiting three critical-mode shapes (one distortional and two global, due to the lack of symmetry). Moreover, an investigation conducted to assess the possible occurrence of "secondary distortional-bifurcation D-G interaction – SDI" ($M_{crG} << M_{crD}$) is also presented and discussed. In order to clarify the surprising behaviour of the beam undergoing a SGI, an additional beam is analysed, exhibiting a "pure" distortional post-buckling behaviour (i.e., involving no coupling phenomenon). The GBT-based results provide the evolution, along given equilibrium paths, of the beam deformed configuration (expressed in modal terms), relevant displacement profiles and modal participation diagrams. The knowledge acquired has visible impact on the development of rational design rules for CFS beams affected by D-G interaction.

1. Introduction

The open thin-walled cross-sections commonly exhibited by cold-formed steel (CFS) members (columns, beams and/or beam-columns) make them structurally very challenging, due to their high susceptibility to several instability phenomena, involving individual (local, distortional, global – L, D, G) and/or coupled (L-D [1–3], L-G [4], D-G [5,6] or L-D-G [7,8]) buckling behaviours. The presence of distortional buckling and associated interactions makes the design of such members a very demanding task (much more that of hot-rolled steel members), since interaction effects may occur even when the corresponding critical buckling loads/moments are visibly apart – when the competing buckling loads/moments are relatively close, the coupling phenomenon unavoidably occurs and corresponds to the so-called "true interaction". Moreover, assessing the structural behaviour of the aforementioned members requires acquiring in-depth knowledge about the mechanics associated with the coupling between the three above buckling modes, genuine geometrically non-linear features that influence considerably the member post-buckling behaviour. This work deals with CFS lipped channel beams experiencing D-G interaction, a topic lacking research since, to the authors' best knowledge, very few available publications have been devoted to its understanding – the only existing studies concern (i) experimental ("4-point bending") and numerical investigations on stainless steel LC beams [9,10] and (ii) the recent shell finite element investigation on the relevance, strength and direct strength design of CFS LC beams [11]. Thus, a significant research effort is needed before efficient design rules can be established for beams undergoing D-G interaction – this paper, which aims to contribute towards fulfilling this objective, focuses on studying the (non-linear) mechanics underlying the post-buckling behaviour of "intermediate-to-long" lipped channel beams affected by this coupling phenomenon.

In the last few years, Generalised Beam Theory (GBT) has emerged as a very promising alternative to assess rigorously the non-linear behaviour of prismatic thin-walled members (and also frames) in a remarkable structurally clarifying manner. In fact, GBT-based analyses make it possible to unveil and quantify the contributions of the various "deformation patterns (modes)" to the member response, thus enabling the acquisition of much deeper insight on the mechanics underlying

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that same response – this is why they are an ideally suited tool to investigate complex coupling phenomena [12–14], similar to the one addressed in this work. Members with well defined loading and support conditions are best suited for this type of studies, since it becomes much easier to interpret the complex behavioural features typically found in mode interaction phenomena. Thus, the objective of this paper is to present and discuss the results of a GBT-based (numerical) investigation on the elastic post-buckling (non-linear) behaviour of simply supported LC beams affected by D-G interaction under uniform major-axis bending – the GBT results are obtained by means of geometrically non-linear imperfect analyses (GNIA) based on beam finite element formulation recently derived and numerically implemented [15]. Two LC beam geometries are considered, each exhibiting a given D-G interaction type, namely (i) "true D-G interaction" ($M_{crG} \cong M_{crD}$) and (ii) "secondary global-bifurcation interaction – SGI" ($M_{crD} < < M_{crG}$) – while the former is analysed with three critical-mode initial geometrical imperfections (IGI), namely one distortional and two global (due to the lack of symmetry), the latter is analysed only with a critical (distortional) IGI. In order to provide a more clear explanation for a few quite surprising results obtained from the analysis of the beam undergoing SGI, and additional beam non-linear behaviour is investigated – that of a beam exhibiting a "pure" distortional post-buckling behaviour. Moreover, the possible occurrence of "secondary distortional-bifurcation interaction – SDI" ($M_{crG} < < M_{crD}$) is also investigated. The GBT-based results consist of beam equilibrium paths, the evolution, along those paths, of the beam deformed configuration (expressed in modal form) and relevant displacement profiles, and also modal participation diagrams. It is expected that the in-depth knowledge acquired in the course of the investigation reported in this work will have a meaningful impact on the search for an efficient design approach aimed at estimating the load-carrying capacity of CFS beams affected by D-G interaction.

2. Buckling analysis – beam geometry selection

In order to study the mechanics underlying the geometrically non-linear behaviour of simply supported LC steel ($E = 210$ GPa, $\nu = 0.3$) beams experiencing D-G interaction, it is indispensable to begin by selecting beam geometries (cross-section dimensions and lengths) ensuring the occurrence of this coupling phenomenon. This selection can be conducted exclusively on the basis of GBT buckling results. For the sake of simplicity, it was decided to base on a single GBT nodal discretisation the (i) buckling results presented in this section and (ii) post-buckling results addressed in Sections 3 and 4 (naturally, such discretisation is unnecessarily refined in some cases). It involves 17 nodes (6 natural and 11 intermediate – 3 in the top/compressed flange, 7 in the web and 1 in the bottom/tensioned flange), leading to the following deformation modes: (i) 19 conventional (4 global, 2 distortional and 13 local – **1**–**19**), (ii) 16 shear (5 global and 11 local – **20**–**35**, (iii) 16 linear transverse extension (1 global isotropic, 4 global deviatoric and 11 local – **36**–**51**), and (iv) 16 quadratic transverse extension modes (**52**–**67**), totalling 67 deformation modes – the configurations of all of them are displayed in Fig. 1[1].

Three LC beam geometries are selected on the basis of their M_{crG} and M_{crD} values, namely (i) one undergoing "true D-G interaction" ($R_{GD} = M_{crG}/M_{crD} \approx 1.00$), (ii) one prone to the occurrence of "secondary global-bifurcation D-G interaction" ($R_{GD} \approx 2.00$) and (iii) one possibly susceptible to the occurrence of "secondary distortional-bifurcation D-G interaction" ($R_{GD} \approx 0.50$). While the first two beams always experience D-G interaction, due to either (i) the closeness between M_{crG} and M_{crD} or (ii) the moderate beam distortional post-critical strength, this coupling phenomenon may not occur in the third beam, due to the well-known small global post-critical strength.[2] Note that, in all the beams selected, the critical local buckling moment (M_{crL}) is large enough to preclude the occurrence of L-D-G interaction. While Fig. 2(a$_1$)–(a$_3$) provide the variation of M_{cr} with the beam length L (logarithmic scale), for the beams selected, Fig. 2(b$_1$)–(c$_3$) display their critical mode.

shapes and associated GBT modal amplitude functions[3] (used to define all the IGIs considered in this work). Fig. 2(a$_1$) depicts also the length selected to ensure nearly coincident M_{crD} and M_{crG} ("true D-G interaction"): $L_{GD} = 205$ *cm* – Fig. 2(b$_1$)–(c$_1$) show the global and distortional buckling mode shapes and associated modal amplitude functions concerning this beam. Similarly, Fig. 2(a$_2$) depicts also the lengths selected to ensure (i) SGI ($R_{GD} \approx 2.00 – L_{GD} = 200$ *cm*) and (ii) a "pure" distortional post-buckling behaviour ($L_D = 40$ *cm* – one has $M_{crD} =$ 2435.6 kNcm $< M_{crL} =$ 6635 kNcm $< < M_{crG} =$ 129,066 kNcm) – Fig. 2(b$_2$)–(c$_2$) show, for both beams, the critical distortional buckling mode shape and associated modal amplitude functions. Finally, Fig. 2(a$_3$) depicts also the length selected to possibly ensure SDI ($R_{GD} \approx 0.50 – L_G = 450$ *cm*) – Fig. 2(b$_3$)–(c$_3$) show the global and distortional buckling mode shapes and associated modal amplitude functions concerning this beam. The observation of these GBT buckling results leads to the following remarks:

(i) For lengths close to the transition between "intermediate" and "long" beams,[4] the critical global buckling modes always has relevant contributions from modes **3** + **4** + **5** (mostly), as shown in Fig. 2(a$_1$) + (b$_1$), and tiny ones from modes **2** + **6**, as depicted in Fig. 2(b$_1$) – all displacement profiles are single half-wave sinusoids. Naturally, longer beams ($L > 400$ *cm*) exhibit a truly global buckling behaviour (modes **3** + **4**).
(ii) All critical distortional buckling mode shapes (Fig. 2(c$_1$) + (b$_2$) + (c$_2$) + (c$_3$)) exhibit dominant contributions from modes **5** + **6** and very tiny (but not negligible) ones from local modes **8** + **9**. In the selection procedure, beam geometries associated with odd and even critical half-wave numbers (n_D) were intentionally chosen, since the corresponding mechanical behaviours are distinct (see Section 4): the $R_{GD} = 1.00$ and $R_{GD} = 2.00$ beams exhibit critical buckling modes with $n_D = 6$ and $n_D = 5$, respectively.[5]

3. Distortional post-buckling behaviour

The results addressed presented in the following sections (including this one) were obtained by means of the GBT formulation recently developed by the authors [15]. Before addressing the beams undergoing D-G interaction, which is done in Section 4, it is important to present and briefly discuss results concerning a beam that exhibits a "pure" distortional post-buckling behaviour – such beam is hereafter termed "D beam". Such discussion is essential to interpret the response of the beam affected by SGI (see Section 4.2). Fig. 3(a$_1$)–(a$_2$) show the equilibrium paths M/M_{crD} vs. $(v + v_0)/t$, where (i) v is the mid-span top flange-lip corner vertical displacement (v_0 is its initial value) and (ii) t is the cross-section wall thickness, of D beams containing critical-mode IGI involving outward top flange-lip motions and with amplitudes $0.10\,t$ and $0.15\,t$, respectively. As discussed below, the post-buckling behaviours of these two beams are significantly (and surprisingly)

[1] For a better grasp on the designations of the various deformation mode families and sub-families the interested reader is referred to [16].

[2] Note that changing the beam loading and/or end support conditions may alter this assertion.

[3] The GBT modal amplitude functions depend only on the longitudinal coordinate and are the unknowns of the problem/analysis under consideration (buckling, in this section, and post-buckling, in Sections 3 and 4 – in the latter case, these functions depend also on the equilibrium state). They describe the longitudinal (along the member length) variation of the amplitude of each deformation mode participation.

[4] "Intermediate" and "long" are commonly used designations to indicate that the member (beam, in this investigation) critical buckling mode is distortional or global, respectively.

[5] As will be shown in Section 4.3, the $R_{GD} = 0.50$ beam n_D value is not relevant.