Thin-Walled Structures 124 (2018) 72–80

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

The Brazier effect for elastic pipe beams with foam cores

A. Luongo*, D. Zulli, I. Scognamiglio

International Center for Mathematics & Mechanics of Complex Systems, M&MoCS, University of L'Aquila, 67040 Monteluco di Roio, AQ, Italy

ARTICLE INFO

Keywords:
Pipe beam
Foam core
Brazier effect
Ovalization
Instability

ABSTRACT

Pipe beams are considered under the action of bending. In case of elastic material, an analytical model of a one-dimensional beam is introduced, where the internal constraint between ovalization and bending curvature is deduced from mechanical considerations. Then, the softening moment-curvature relationship, able to describe the Brazier effect, is evaluated applying equilibrium equations in the nonlinear field. The same model is extended to the case where a structural foam is present as core of the pipe. The contribution of the core is analyzed in terms of its action in preventing instability phenomena. Finally, a model of lumped ovalization is discussed.

1. Introduction

Tubular thin-walled beams are essential assets in many industrial and civil applications, from gas and oil distribution networks, nuclear plants, aerospace structures, waterworks and many others. The evaluation of their bearing capacity appears as a crucial step in the design but, on the other hand, it is well-known that conventional beam models typically fail at that, due to the consequences of the usually significant deformation of the cross section. In particular, a possible source of instability is the Brazier effect [1], which is related to the ovalization of empty tubes under bending, giving rise to a nonlinear softening behavior of the structure and the occurrence of catastrophic limit points, even in elastic regime. This phenomenon is analyzed in [2], where a variational approach in small-strain nonlinear elastic regime is used to model the combined effects of cross-section deformation and localized longitudinal buckling in case of pure bending of thin-walled tubes with circular cross-sections. In [3], the instability analysis under bending effect is addressed to the case of very short cylinders, taking into account imperfections as well. In [4], long thin elastic tubes with possible initial curvature are considered in the framework of finite elements analysis, with the aim of comparing the triggering of both buckling and ovalization instability, as well as tracing the post-buckling paths. Critical failure in wind turbine blades is analyzed in [5], where it is shown how the Brazier pressure may have a significant impact in the mechanical behavior of such kind of structures. In [6], bending collapse behavior of thin-walled circular tubes is addressed, after deriving the relationship between the applied moment and the bending angle, and then generating simplified tube models with different cross-sections and materials. In [7], a one-dimensional continuum endowed with structure is proposed to analyze the pure flexure problem of rods, finding bifurcation conditions which can describe the Brazier instability of thin-walled tubes. In [8], anisotropic materials are considered in formulating a beam model of cylindrical tube, and use of the variational-asymptotic method is made to obtain asymptotically correct solutions reproducing the Brazier limit-moment instability. In [9], single- and double-walled elastic tubes are analyzed under a pure bending condition, and the modification of the ovalization intensity and of the Brazier limit-moment due to the presence of the multiple layers is evaluated.

Sometimes, the presence of a soft core, possibly made of foam materials, can be used to improve the performance of the pipes under bending instability. As an example, the presence of soft elastic cores in thin-walled cylindrical structures is considered in [10], in which attention is paid to structures as suggested by nature, where foam-like cellular cores fill, e.g., plant stems or hedgehog spines, in order to obtain inspiration to increase the mechanical efficiency of engineering structures. In [11,12], analytical and experimental models are used to find the optimum design of thin-walled, cylindrical shells with compliant cores subjected to uniaxial compression and bending.

More specifically about the foam material properties, as well as its interaction with the skin structure, composite sandwich panels developed for marine applications with PVC foam core are analyzed in terms of flexural behavior under quasi static load in [13], while polymeric foam composites for aerospace industry are considered in [14]. Very soft polyurethane foam cores are studied and characterized in [15], even in cases of different fabrication angles.

In [16] a locally deformable one-dimensional beam model was used to describe effects of bending on thin-walled members, after identifying the nonlinear elastic response function from a corresponding three-dimensional fiber model. Drawing inspiration by [17], where distortion-constrained thin walled beam models are formulated to describe,

* Corresponding author.
E-mail address: angelo.luongo@univaq.it (A. Luongo).

https://doi.org/10.1016/j.tws.2017.11.053
Received 18 May 2017; Received in revised form 27 November 2017; Accepted 30 November 2017

among others, effects of bending, here an internally constrained pipe beam under bending and able to be filled with structural foam is considered. The aim is to derive a suitable bending moment-curvature relationship, which is obtained with reference to a segment of beam in uniform bending, but which could be assumed as valid for a nonuniform bending condition too, as it is typically done in engineering applications. A constraint between the ovalization, assumed as occurring in a prescribed oval shape, and the imposed bending curvature is obtained using mechanical considerations. Then, the corresponding softening moment-curvature relationship is obtained through equilibrium equations in the nonlinear field, in order to describe the Brazier effect. The model is then extended to the case of filled pipes, where a structural foam is used as core, to increase the stiffness and reduce the possibility of instability occurrence. Even if structural foams can present complicated and rich response laws, showing nonlinear elastic, viscous and plastic behavior with plateau regions (see [15,18,19]), here they are assumed to remain in linear elastic field, postponing the accounting of nonlinear material effects, in particular plasticity, to future works.

The paper is organized as follows: In Section 2 the analysis of empty pipes under bending is carried out; in Section 3, a foam core is considered and the model extended consistently; in Section 4 an application on a simple structure is carried out, suggesting the introduction of a simplified model. Finally, in Section 5, some conclusions are drawn.

2. The empty pipe

2.1. The mechanical interpretation of the Brazier effect

Here the Brazier effect is referenced through a mechanical interpretation directly deduced by [1]. A beam with a thin annular cross section and length l is considered. The beam axis is spanned in the direction identified by the unitary vector $\mathbf{a}_1$. The average radius of the cross section is R, its uniform thickness is $b \ll R$, and $\mathbf{a}_2$, $\mathbf{a}_3$ are orthogonal unitary vectors laying on it (see Fig. 1-a,b). The beam is constituted by linear elastic material of Young's modulus E_s, where the subscript s stands for "skin". A generic longitudinal fiber of the beam is identified in polar coordinates by the phase φ on the cross section (or equivalently by the abscissa $c = R\varphi$), therefore its initial distance from the axis $\mathbf{a}_3$ is $y = R\sin(\varphi)$. A finite but small segment of beam of length $\Delta s \lll l$ is considered and imaged under the action of uniform bending. The deformed shape of the beam segment is related to the imposition to the axis line of a uniform (and small) curvature κ about $\mathbf{a}_3$ (see Fig. 2-a); consistently with Saint Venant theory, a longitudinal stress occurs. In particular, the generic fiber, represented separately in Fig. 2-b, assumes radius of curvature equal to $\frac{1}{\kappa} + y \simeq \frac{1}{\kappa}$, and is subjected to the longitudinal stress σ_s. Note that nonlinear effects have to be considered in evaluating the contribution of σ_s, overtaking in some sense Saint Venant theory, namely σ_s is not generally directed as $\mathbf{a}_1$ but it is perpendicular to deformed (and still planar) cross section (or, equivalently, tangent to the deformed fiber, as in Fig. 2-b, since shear strain is null); moreover, in the same framework, the intensity of σ_s in a point of the cross section shall be evaluated as a linear function of the distance of the point itself, in the deformed configuration, from $\mathbf{a}_3$, which is the neutral axis. Following the Mariotte formula [20], the equilibrium of the longitudinal fiber under the action of σ_s is guaranteed by the existence of a radial pressure of intensity

$$p = \sigma_s b\kappa \tag{1}$$

On the other hand, the prescribed pressure is actually not present, therefore a contrary pressure of the same intensity must be imaged as applied in correspondence of the fiber itself (see Fig. 3-a), in order to vanish the total pressure. Generalizing the result for any fibers, the contrary pressure has intensity depending on the distance of the fiber from $\mathbf{a}_3$, as in Fig. 3-b. In the perspective of linear kinematic equations in the transverse direction, the pressure is assumed to act along $\mathbf{a}_2$; it is responsible for the ovalization of the cross section.

2.2. Deformation of the cross section and application of the virtual work theorem

The cross section is supposed to displace to an assumed shape of oval, where the change in length of the semi-axes is referred to as α, considered uniform along the segment of beam (Fig. 3-c). In particular, the displacement of the trace point of the longitudinal fiber identified by the phase φ is

$$\mathbf{u}(\varphi) = u(\varphi)\mathbf{a}_t(\varphi) + v(\varphi)\mathbf{a}_n(\varphi) \tag{2}$$

where $\mathbf{a}_t(\varphi)$, $\mathbf{a}_n(\varphi)$ are the tangent and (internal) normal unitary vectors at the abscissa c; the components u, v are considered as

$$u(\varphi) = \alpha\psi_t(\varphi) \tag{3}$$

$$v(\varphi) = \alpha\psi_n(\varphi)$$

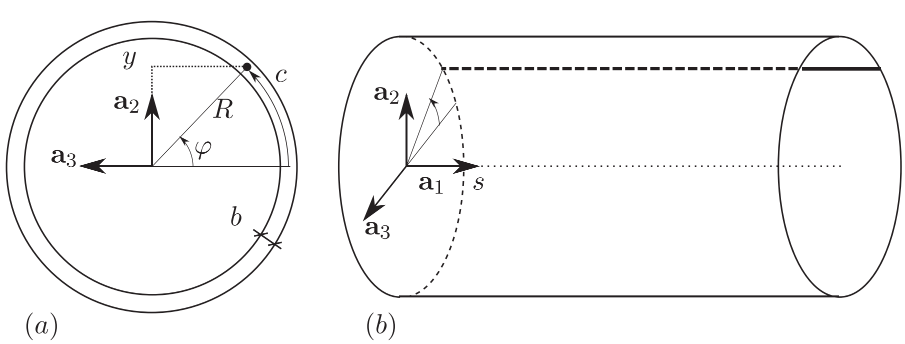

Fig. 1. Pipe beam: (a) cross section, (b) beam with a generic longitudinal fiber highlighted.

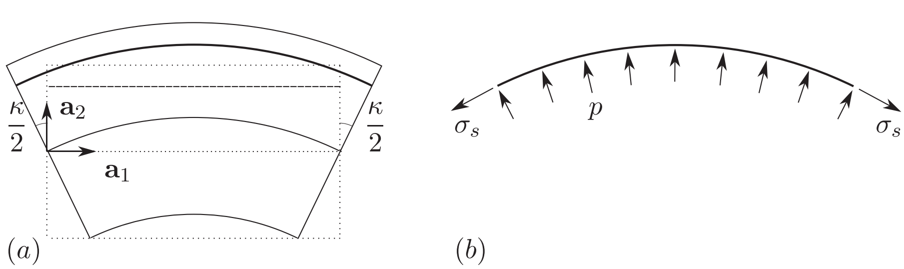

Fig. 2. Deformation of the beam segment under pure bending (a). Stress on the generic fiber plus radial pressure (b).