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Elastic collapse of thin long cylindrical shells under external pressure

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A B S T R A C T

This paper investigates local elastic buckling of thin long cylindrical shells under external pressure. Based on Donnell's and Sanders' theories of thin shells and von Karman nonlinearity assumptions, the potential energy is derived. The buckling load and curves of the static equilibrium path are obtained using the Ritz method. The results are validated with the existing ones in the literature. Furthermore, the case where the pressure is perpendicular to the deformed state is compared with a dead loading. It is demonstrated that the former yields a lower critical pressure in both shell theories.

1. Introduction

Cylindrical shells are widely used in many engineering structures such as aircraft, spacecraft, nuclear reactors, cooling towers, pressure vessels, pipelines, and offshore platforms. Since these structures are usually thin, buckling is the controlling failure mode. Hence, it is essential to understand the buckling behavior of cylindrical shells properly and establish appropriate design methods. In this study, the attention is dedicated to long cylindrical shells with diameter to thickness ratio ($\boldsymbol{D}/\boldsymbol{t}$) larger than 40, for which collapse occurs by means of an elastic flattening before the pipe material reaches its yield strength [1].

Brayan [2] was the pioneer in deriving an expression for the collapse pressure of thin long tubes free from any form of end constraints. Timoshenko and Gere [3] presented an equation for elastic buckling of long cylindrical shells subject to external pressure. They investigated the buckling mechanism of a ring subject to external pressure. Based on the plane strain assumption, they further extended their solution to cylindrical shells with free edges as well as to infinitely long cylindrical shells. They presented the buckling pressure, p_{cr} of the cylindrical shell as follows [3]:

$$p_{cr} = \frac{E}{4(1-\nu^2)}\left(\frac{t}{R}\right)^3 \tag{1}$$

where R, t, E, and ν denote the radius, the thickness, the Young's modulus, and the Poisson's ratio, respectively. The formula presented by Timoshenko and Gere is widely used for the design of long cylindrical shells as in DNV-OS-F101 [4] to calculate the elastic collapse pressure of the pipe.

Much work has been dedicated to determining the collapse pressure of a circular cylindrical shell with finite length (less than critical length) under external pressure (see [2–15], and the references therein). The critical length is the minimum length of a cylindrical shell for which the collapse pressure is independent of the end boundary conditions and any further increase in length. The following formula was first proposed by Southwell [5] for the critical length:

$$L_c = Kd\sqrt{d/t} \tag{2}$$

where d is the diameter of the cylindrical shell and $K = (4\sqrt{6}\pi\sqrt{1-\nu^2})/27 = 1.11 (for \nu = 0.3)$. Cook's experimental tests [5] verified the formula but proposed a value of $K = 1.73$ instead. Von Mises [5,6] derived an equation for the collapse pressure of simply supported thin short tubes exposed to lateral pressure. Later, he [5,7] extended his work to include both lateral pressure and axial load. Based on a throughout review on the relevant theoretical and empirical instability formulas in the literature, Windenburg and Trilling [5] proposed a formula for calculating the collapse pressure of thin cylindrical shells with a length less than the critical one subject to uniform external pressure. Sturm and O'Brien [8] determined the collapse pressure for simply supported and clamped thin cylindrical shells subject to uniform external pressure. In their study, they accounted for the plastic behavior, out of roundness of the cylinder, and ring stiffening effects. Sturm [9] studied the behavior of thin-walled tubes under uniform external pressure experimentally and compared his results with the existing theories. Yamaki and Otomo [10] performed experimental studies on the post-buckling behavior of circular cylindrical shells subject to hydrostatic pressure by using test specimens with a radius equal to 100 mm, a length ranging from 23 mm to 165 mm, and a thickness equal to 0.25 mm. Based on the nonlinear Karman-Donnell's equations, Shen and Chen [11] studied the effects of external pressure on buckling and post-buckling behavior of cylindrical shells with clamped edges

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using boundary layer theory and a singular perturbation method. Based on Kirchhoff-Love assumptions, Cheung and Zhu [12] investigated the post-buckling of circular cylindrical shells with finite length under uniform external pressure using the spline finite strip method. Dyau and Kyriakides [13] experimentally and numerically addressed the collapse mechanism in cylindrical shells under external pressure where both geometric and material nonlinearities were considered. Based on Flügge's stability equations, Vodenitcharova and Ansourian [14] studied the buckling behavior of circular cylindrical shells subject to external pressure. They showed that their length dependent model gave a slightly lower buckling pressure than that given by Timoshenko and Gere [3] (see Eq. (1)). Pinna and Ronalds [15] derived the buckling load of cylindrical shells with different boundary conditions under hydrostatic load via an eigenvalue analysis. More recent investigations on the subject have considered other parameters affecting the buckling mechanism such as initial imperfection [16,17], thickness variation [18–21], and time dependent external pressure [22].

In comparison with the available analyses on finite length cylindrical shells, the subject of local buckling of long cylindrical shells under external pressure has attracted less attention. In infinitely long cylindrical shells under external pressure, local buckling occurs and a certain length along the longitudinal axis of the shell experiences radial deflection, which is clearly observed in full-scale test specimens [23] (see Fig. 1). Therefore, extending the solution for buckling of a ring to the case of a long cylindrical shell using the plane strain assumption is questionable, since it is assumed that the whole cylindrical shell deforms into an elliptical cylinder. Fraldi and Guarracino [24] also pointed out this deficiency and presented an analytical formulation for the capacity load of circular rings under external pressure, which accounts for the onset of plasticity and geometric imperfections. Xue and Hoo Fatt [25] studied elastic buckling of a non-uniform long cylindrical shell subject to external pressure and proposed a set of formulas for symmetric and anti-symmetric buckling modes. Later, based on Donnell-Mushtary's shell theory and using the Ritz method along with some simplifying assumptions, Xue [26] managed to present a formula for the buckling pressure of long cylindrical shells under external pressure. The buckling pressure predicted by Xue's formula [26] was 33% higher than the one predicted by Timoshenko and Gere in Eq. (1). In order to overcome this discrepancy, Xue et al. [27] considered the influence of the initial curvature of the cylindrical shells on Karman-Donnell's equations (they considered an extra term equal to $1/R - 1/(w-R) \approx -w/R^2$ in the equation for the circumferential curvature) and showed that taking the initial curvature into account, has a significant influence on the load carrying capacity and is in agreement with Timoshenko's result (Eq. (1)).

In the present study, based on the classical Donnell's and Sanders' theories of shells and von Karman nonlinear strain-displacement relations, the elastic collapse pressure and post buckling behavior of thin long circular cylindrical shells under uniform external pressure is investigated adopting the Ritz method. Most importantly, it is shown that taking into account the perpendicularity of pressure to the deformed state (instead of a dead load), decreases the collapse pressure and achieves a closer agreement with the empirical results as well as that of Timoshenko's. In addition, the static equilibrium path and mode shapes are studied.

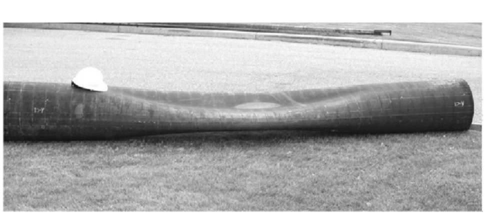

Fig. 1. Photograph of a 20-in., × 80 pipe collapsed under external pressure (courtesy C-FER Technologies) [23].

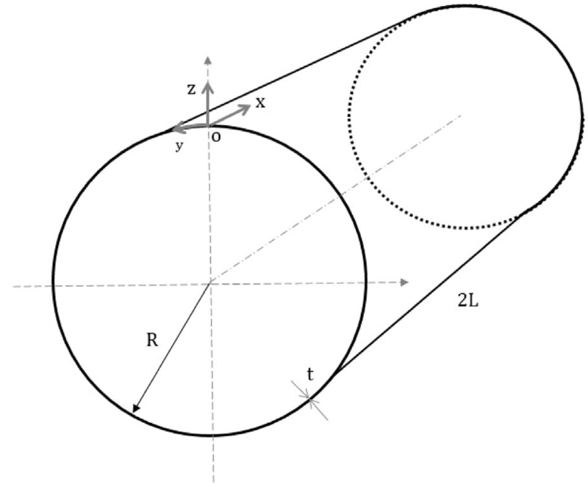

Fig. 2. The geometry of the shell and the curvilinear coordinate system.

2. Formulation

A circular cylindrical shell of mean radius R, thickness t, and length $2L$ is considered. The geometry of the shell and the curvilinear coordinate system (x, y, z) considered at the mid-surface of the shell is shown in Fig. 2. The displacements of an arbitrary point at the mid-surface of the shell are denoted by u, v, and w in the axial (x), circumferential (y), and radial (z) directions, respectively.

2.1. Strain-displacement relations

The strains and the changes in curvature of the mid-surface in the classical Donnell's and Sanders' theories of shells are as follows [28,29]:

$$\varepsilon^0{}_{xx} = u_{,x} + \frac{1}{2}w_{,x}{}^2; \quad \varepsilon^0{}_{yy} = v_{,y} + \frac{w}{R} + \frac{1}{2}(w_{,y} - \lambda\frac{v}{R})^2;$$
$$\varepsilon^0{}_{xy} = \frac{1}{2}(v_{,x} + u_{,y} + w_{,x}(w_{,y} - \lambda\frac{v}{R})) \tag{3}$$

$$\kappa_{xx} = -w_{,xx}; \quad \kappa_{yy} = -w_{,yy} + \lambda\frac{v_{,y}}{R}; \quad \kappa_{xy} = -w_{,xy} + \frac{\lambda}{4R}(3v_{,x} - u_{,y}) \tag{4}$$

where λ is a coefficient taking on the value of either zero or one. In order to have a Donnell's or Sanders' theory, the coefficient is set, respectively, $\lambda = 0$ or $\lambda = 1$. The strain components ε_{xx}, ε_{yy}, and ε_{xy} at an arbitrary point of the shell are given by:

$$\varepsilon_{xx} = \varepsilon^0{}_{xx} + z\kappa_{xx}; \ \varepsilon_{yy} = \varepsilon^0{}_{yy} + z\kappa_{yy}; \ \varepsilon_{xy} = \varepsilon^0{}_{xy} + z\kappa_{xy} \tag{5}$$

where z is the distance from the arbitrary point of the shell to the mid-surface.

2.2. Stress-strain relations

Based on the classical thin shell theory (where the transverse normal stress is neglected), the stress-strain relations in a plane stress state for a homogeneous isotropic elastic body are [28]:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & 1-\nu\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{bmatrix} \tag{6}$$

The stress and bending moment resultants for a thin cylindrical shell are defined as:

$$\{N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy}, M_{xy}\} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{xx}z, \sigma_{yy}z, \sigma_{xy}z\}dz \tag{7}$$

Upon substituting Eqs. (5) and (6) into Eq. (7), the stress and