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GBTUL 2.0 − A second-generation code for the GBT-based buckling and vibration analysis of thin-walled members

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ABSTRACT

This paper presents the fundamentals and illustrates the application and potential of the recent 2.0 release of the software GBTUL – a computer program developed by the authors and made available as freeware on the website of the Department of Civil Engineering of the University of Lisbon. The program is based on Generalised Beam Theory (GBT), a bar theory accounting for cross-section in-plane and out-of-plane (warping) deformation, and performs linear buckling and undamped free vibration analyses of prismatic thin-walled members. Its domain of application is much wider than that of the previous release (1.0β), making it possible to analyse single or multi-span members (i) with various support conditions, namely those due to discrete bracing systems, (ii) exhibiting arbitrary (open, closed or "mixed") flat-walled cross-sections and (iii) acted by fairly general loadings, including concentrated and/or distributed transverse forces applied away from the member shear centre axis. After providing a brief overview on the GBT fundamentals, the program capabilities and innovative aspects are addressed, and its application is illustrated by means of a few relevant numerical examples. Moreover, the program Graphical User Interface is described and the procedures and/or options associated with its main commands are mentioned.

1. Introduction

It is well-known that, in general, the structural behaviour of thin-walled members can be highly influenced by complicated non-linear phenomena involving cross-section in-plane and out-of-plane (warping) deformation [1]. Therefore, the safe design of such members requires the use of complex mechanical models, which cannot be derived from classical beam theories and whose numerical implementations are often far from straightforward, require time-consuming computations and lead to results difficult to interpret. Moreover, in the context of thin-walled structures, the most modern design codes include provisions concerning ultimate and serviceability limit states that require in-depth knowledge about the member buckling and vibration behaviour. For instance, it is worth mentioning the increasingly popular and universally accepted Direct Strength Method (DSM – *e.g.*, [2]), which application is based on (i) the identification of the nature of the relevant buckling modes (local, distortional or global) and (ii) calculation of the associated buckling loads and/or moments. In the particular case of cold-formed steel members, the performance of this task requires using either (i) Generalised Beam Theory (GBT − *e.g.*, [3–5]), (ii) the Constrained Finite Strip Method (cFSM – *e.g.*, [6–8] and see [9] for a comparison with GBT) or the Shell Finite Element Method (SFEM – *e.g.*, [10]). Although the last method is undoubtedly the most versatile, it is also very time-consuming and, most importantly, does not allow for a straightforward identification of the buckling mode nature. In order to remedy these drawbacks, in the context of buckling analysis, (i) GBT deformation modes have been employed to constrain the SFEM displacement field, thus leading to results identical to those obtained with GBT buckling analyses [11], (ii) SFEM solutions have been constrained according to the cFSM principles [12,13], (iii) SFEM solutions have been projected into the GBT mode space using the orthogonality properties of the GBT modal matrices [14] and (iv) the SFEM has been blended with GBT to improve the accuracy of the latter in geometrically and/or materially non-linear problems [15]. On the other hand, the only cFSM-based software currently available (CUFSM 4.05, developed at Johns Hopkins University by Li and Schafer [16]) can handle exclusively members acted by uniform internal force/moment diagrams and exhibiting fairly standard support conditions − moreover, it does not perform vibration analyses, essential for serviceability limit state checks.

Generalised Beam Theory (GBT) is an elegant, insightful and computationally efficient approach to perform structural analyses of thin-

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walled members. Its main distinctive feature is the fact that the displacement field is expressed as a combination of structurally meaningful *cross-section deformation modes*, whose amplitudes (the problem unknowns) vary along the member length. This technique, originally proposed and considerably developed by Schardt [17], has been continuously updated in the last decade, mostly due to the research efforts carried out at the University of Lisbon (*e.g.*, [3–5]) – for instance, GBT formulations have been developed to perform first-order [18,19], buckling [20–24], vibration [25,26], dynamic [27] and post-buckling [28–30] analyses involving mostly prismatic members (an application to tapered members was recently reported [31]), but also frames and trusses. Quite recently, GBT formulations for physically non-linear problems were also developed and numerically implemented [32–36]. Moreover, other researchers have also contributed to developments in this field (*e.g.*, [37]).

In the context of the abovementioned research activity, the authors developed a GBT-based user-friendly computer program named GBTUL (acronym for "GBT at the University of Lisbon" [1]) and its first release (GBTUL 1.0β) has been freely available online, since 2008, on the website of the Civil Engineering Department of the University of Lisbon [38]. However, this first version of GBTUL was only applicable to isolated (single-span) members (i) with open cross-sections (*i.e.*, no closed cells allowed), (ii) acted by few types of loading (transverse loads could only be applied at the shear centre axis) and (iii) exhibiting a quite limited number of support conditions, specified only at the two end cross-sections.

This paper presents the most recent version of the upgraded 2.0 release of GBTUL [39] and illustrates its application and potential to perform linear buckling and undamped free vibration analyses of thin-walled members − note that GBTUL 2.0 was first released in 2014 and has been continuously improved since then (the most recent version is from the beginning of 2016). The program incorporates the latest GBT developments, which make it possible to overcome several of the aforementioned limitations of its predecessor. Among the new capabilities, the following ones should be highlighted: (i) systematic and hierarchic determination of the deformation modes for arbitrary flat-walled cross-sections (*i.e.*, cross-sections combining arbitrarily closed cells and open branches), which is done through the implementation of the most recent cross-section analysis procedure [40,41], (ii) the consideration of general pre-buckling stress distributions, including shear and transverse normal stresses (which play a key role in capturing the effects of non-uniform bending and/or the height of transverse loads [22]), (iii) the presence of arbitrary support conditions, including intermediate supports (multi-span beams and bracing systems) and (iv) the consideration of concentrated or distributed localised masses and/or elastic supports. Finally, the quality of the Graphical User Interface (GUI) was considerably improved, leading to a much better input/output processing and visualisation.

2. Generalised Beam Theory – brief overview

As mentioned above, GBT is a one-dimensional bar theory that expresses/discretises the member deformed configuration as a linear combination of cross-section deformation modes multiplied by their amplitude functions. Any GBT-based structural (buckling or vibration) analysis follows the general procedure depicted in Fig. 1 – the four main steps are termed (i) Cross-Section Analysis, (ii) Deformation Mode Selection, (iii) Member Analysis and (iv) Solution. A very brief overview of GBT is presented next – more detailed accounts can be found in the literature (*e.g.*, [3–5,17,40]).

Consider the prismatic thin-walled member with the arbitrary cross-section depicted in Fig. 2(a), in which local coordinate systems $x - s - z$ are adopted at the walls, as shown in Fig. 2(b). In GBT, the wall mid-plane axial, transverse and normal displacement components – $u(x, s)$, $v(x, s)$ and $w(x, s)$ – are given by

$$u(x, s) = u_k(s)\varphi_{k,x}(x) \qquad v(x, s) = v_k(s)\varphi_k(x) \qquad w(x, s) = w_k(s)\varphi_k(x), \tag{1}$$

where (i) $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the mid-line functions defining cross-section deformation mode k (or "GBT mode k"), (ii) $\varphi_k(x)$ or $\varphi_{k,x}(x)$ are the amplitude functions describing their variation along the member length, (iii) the commas indicate differentiation, (iv) $1 \le k \le N_d$, where N_d is the total number of deformation modes and (v) the summation convention applies to subscript k. Therefore, the member deformed configuration can be expressed as a sum of contributions from the N_d deformation modes – the contribution of mode k is the product of its mid-line function by the corresponding amplitude functions. Alternatively, Eq. (1) can be written in matrix form as

$$u = \boldsymbol{u}^T\boldsymbol{\varphi}_{,x} \qquad v = \boldsymbol{v}^T\boldsymbol{\varphi} \qquad w = \boldsymbol{w}^T\boldsymbol{\varphi}, \tag{2}$$

where (i) $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are column vectors containing the $u_k(s)$, $v_k(s)$ and $w_k(s)$ functions, respectively, and (ii) $\boldsymbol{\varphi}$ is a column vector containing the corresponding amplitude functions $\varphi_k(x)$.

The member elastic strain energy U, is given by (V is the member volume)

$$U = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}\,dV, \tag{3}$$

where σ_{ij} and ε_{ij} are the stress and strain tensors, and the summation convention applies to all subscripts. Adopting the Kirchhoff-Love hypotheses, a plane stress state is assumed in the walls (with non-null components σ_{xx}, σ_{ss} and τ_{xs}). Moreover, by using Eq. (2) and assuming small strains, together with a St. Venant-Kirchhoff material law, Eq. (3) can be expressed in the form [40]

$$U = \frac{1}{2}\int_L (\boldsymbol{\varphi}_{,xx}^T\mathbf{C}\boldsymbol{\varphi}_{,xx} + \boldsymbol{\varphi}_{,x}^T\mathbf{D}\boldsymbol{\varphi}_{,x} + \boldsymbol{\varphi}^T\mathbf{B}\boldsymbol{\varphi} + \boldsymbol{\varphi}_{,xx}^T\mathbf{E}\boldsymbol{\varphi} + \boldsymbol{\varphi}^T\mathbf{E}^T\boldsymbol{\varphi}_{,xx})dx, \tag{4}$$

where L is the member length and $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{E}$ are $N_d \times N_d$ linear stiffness matrices, associated with several cross-section mechanical properties, namely (i) primary/secondary warping, (ii) transverse extension/flexure, (iii) wall shear distortion/torsion and (iv) membrane/flexural Poisson effects – the analytical expressions for their components are provided in Annex A.

2.1. Cross-section analysis

As shown in Fig. 1, the first step of a GBT structural analysis is the determination of the cross-section deformation modes ($u_k(s)$, $v_k(s)$ and $w_k(s)$ functions) and associated mechanical properties (C_{ik}, B_{ik}, D_{ik} and E_{ik} components), which is done through a systematic procedure termed *Cross-Section Analysis*. The program uses a recently developed version of this procedure, applicable to arbitrary flat-walled members and described in detail in [40,41]. It starts with the specification of the cross-section nodal discretisation. The number, nature and quality of the deformation modes obtained depend on this nodal discretisation, which involves (i) natural intermediate nodes, (ii) natural end nodes and (iii) intermediate (user-defined) nodes (see [40] for more details). The associated N_d deformation modes, which are automatically computed, may be grouped into 3 main families (superscript $(\bullet)^M$ denotes membrane strains): (i) Vlasov modes, for which $\gamma_{xs}^M = \varepsilon_{ss}^M = 0$, (ii) Shear modes, for which $\gamma_{xs}^M \neq 0; \varepsilon_{ss}^M = 0$, and (iii) Transverse Extension modes, for which $\varepsilon_{ss}^M \neq 0$ – these deformation mode families can still be further divided into several sub-families, as described in Table 1.

For illustrative purposes, consider the "arbitrary" cross-section depicted in Fig. 3(a) and the nodal discretisation shown in Fig. 3(b). Fig. 4 displays the in- and/or out-of-plane (warping) shapes of the $N_d = 30$ deformation modes obtained, which are divided as follows:

[1] Previously, this acronym stood for "GBT at the Technical University of Lisbon". The change is due to the fact that the former Technical University of Lisbon and the University of Lisbon have recently merged under the joint name "University of Lisbon".