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A method for the numerical derivation of plastic collapse loads

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ABSTRACT

Two key reference loads: (i) the plastic collapse load and (ii) the elastic buckling load are commonly used to determine the slenderness and hence the resistance of structural steel elements in international design standards. Utilising numerical methods, the plastic collapse loads are typically obtained through a Materially Nonlinear Analysis (MNA) based on small displacement theory (i.e. a first order plastic analysis). However, such analyses can often yield ambiguous or even spurious results due to, for example, the load-deformation paths not reaching a peak value or reaching a peak value but only after unrealistically large deformations, resulting in misleading predictions of plastic collapse loads and mechanisms. In this paper, a standardised means of determining plastic collapse loads from numerical MNA based on attaining a tangent stiffness of 1% of the initial slope of the load-deformation curve is presented. Furthermore, for analyses that terminate prematurely, an extrapolation technique to predict the full load-deformation paths and hence estimate the plastic collapse load is proposed. The accuracy and practicality of the proposed approach over existing methods is illustrated for a wide range of structural scenarios, with an emphasis on structural elements under concentrated transverse forces.

1. Introduction

Structural design codes commonly employ design methods based on two key reference loads: (i) the plastic collapse load and (ii) the elastic buckling load, from which the element slenderness and hence the element resistance can be determined. This approach is often favoured over a full geometrically and materially nonlinear analysis with imperfections (GMNIA) for its simplicity and ease of application (e.g. identification of the most detrimental imperfection patterns is not required by the analyst) [1]. Both of these reference loads can be obtained either through simplified analytical expressions or numerical analyses. For structural elements under uniform loading and with regular geometry and boundary conditions, the former is suitable, while for those with complex non-uniform stress distributions, which may arise due to concentrated transverse loading, numerical analyses are necessary.

For the prediction of the plastic collapse loads of structural elements through numerical methods such as the finite element method, Materially Nonlinear Analyses (MNA) based on small displacement theory using an elastic perfectly-plastic material model (i.e. a first-order elastic-plastic analysis without strain hardening) are typically adopted [2,3]. The plastic collapse loads are assumed to be the load values where the load-deformation curves obtained from MNA become flat, while the elastic critical loads are typically determined through Linear Bifurcation Analyses (LBA). The determination of elastic buckling loads through LBA is generally straightforward. However, in many cases, the determination of plastic collapse loads from MNA is not straightforward since (i) the load-displacement paths obtained from MNA can continue to slowly rise and fail to become flat at the end of MNA or (ii) the analysis can terminate prematurely or (iii) the obtained load-deformation paths can flatten out only after unrealistically large deformations have been reached, with the result that the corresponding load does not reflect the nature of a realistic plastic collapse mechanism.

To obtain the plastic collapse loads in the cases where the load-displacement paths obtained from MNA is still rising at the end of MNA or where the analysis terminates prematurely, a graphical extrapolation technique utilising the load-deformation paths obtained prior to the end of MNA may be used. However, for cases where the load-deformation paths from the MNA flatten out only after unrealistically large deformations, a consistent criterion is necessary for the determination of the plastic collapse loads. For the purpose of determining plastic collapse loads from the MNA of structural elements on a consistent basis, different graphical techniques have been put forward in the literature. Holst et al. [4] and Doerich and Rotter [5] proposed the use of a modified version of the Southwell plot [6], originally developed for the determination of elastic critical loads of steel elements by Horne and Merchant [7], to estimate collapse loads from MNA. The graphs were referred to as Modified Southwell Plots (MS Plots). An alternative

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graphical technique, termed the Convergence Indicator Plot (CIP), was also proposed by Doerich and Rotter [5], which utilises the plastic collapse load projections obtained from the MS Plot to estimate the collapse loads of steel elements. The higher accuracy of the CIP, relative to the MS Plot was illustrated using several examples. However, in these studies, only a limited number of cases, all of which exhibited load-deformation paths with progressively decreasing slopes and without any 'kinks' (which often arise in numerical analyses of steel elements undergoing consecutive localised plasticity in different regions) were considered. Application of the techniques to more problematic cases was not presented. Moreover, the proposed MS Plot and CIP techniques usually require a large number of plastic collapse load projections using a series of regression lines to make accurate estimations of plastic collapse load values. Though this process could be automated, anomalies may arise in some cases, requiring 'personal judgement' of the plots to estimate the plastic collapse loads. Thus, their practicality and consistency may, in some cases, be limited.

In this paper, different graphical techniques for the consistent determination of the plastic collapse loads of structural steel elements from MNA are investigated with an emphasis on elements under concentrated transverse loading. Finite element models of a large number of steel elements are created. Potential problems encountered in the determination of plastic collapse loads from load-deformation graphs are described and the aforementioned techniques proposed for the estimation of the plastic collapse loads are analysed. A new graphical technique based on a tangent stiffness degradation criteria, referred to as the tangent stiffness plot (TS Plot), is developed considering a large range of practical cases. Unlike the MS Plot and CIP techniques, the proposed TS Plot technique does not require a series of projections of the plastic collapse loads for the estimation of its value, leading to a more straightforward and rapid determination of plastic collapse loads from MNA. It is shown that the developed TS Plot is generally both more practical and more accurate than the aforementioned alternative graphical techniques.

2. Finite element model and material stress-strain response

Finite element (FE) models of a large range of steel elements subjected to different loading conditions were developed through the finite element analysis software Abaqus [8] and studied in this paper. Materially Nonlinear Analyses (MNA) based on small displacement theory were carried out on each of the models. In all the considered cases, a bilinear elastic-perfectly plastic stress-strain relationship was utilised with the Young's modulus $E = 200$ GPa and Poisson's ratio $\nu = 0.3$, neglecting strain hardening of the material. An elastic-perfectly plastic material model is invariably adopted in numerical MNA by analysts since the use of a rigid-plastic material model, as adopted in classical plastic theory, can lead to convergence problems when performing MNA using some finite element solution algorithms; this was also observed within the preliminary analyses performed at the onset of this study. Unless otherwise indicated, the yield stress was taken as $f_y = 400$ MPa in all the considered cases. A four-node general purpose reduced integration shell element, referred to as S4R in the Abaqus [8] element library, was adopted in all the numerical simulations. A fine mesh density was used in all the finite element models, with an element size of 5 mm unless otherwise indicated. In line with [5], the MNA were implemented through the modified Riks solver [9,10] without consideration of any geometrical nonlinearities. The generated FE results are used throughout the following sections for the assessment of existing and proposed graphical techniques for the determination of plastic collapse loads from numerical MNA.

3. Potential problems with the determination of plastic collapse loads from MNA

In this section, the aforementioned problems with respect to the

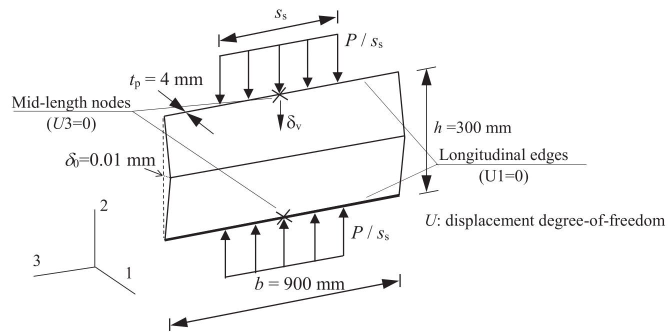

Fig. 1. Transversely loaded steel plate with different loaded lengths s_s.

determination of plastic collapse loads from MNA are illustrated by considering the case of a steel plate subjected to transverse loading with a range of different loaded lengths. The geometry, boundary conditions and loading conditions of the analysed plate are shown in Fig. 1, where s_s is the loaded length, b and h are the width and depth of the plate, t_p is the plate thickness and δ_0 is the imperfection value at the middle of the plate. Taking $b = 900$ mm, $h = 300$ mm, $t_p = 4$ mm and $\delta_0 = 0.01$ mm, different values were adopted for the loaded length s_s. Curves of load P versus vertical displacement δ_v at the middle of the loaded length s_s obtained through MNA are illustrated in Fig. 2. These plates, which are representative of many cases commonly encountered in structural engineering applications, are expected to undergo localised plasticity resulting from local plate bending, in contrast with perfect plates subjected to only membrane yielding. The load deformation curves for three loaded lengths ($s_s = 50$ mm, 400 mm and 700 mm) are shown in Fig. 2. All, after a very high number of increments and excessively high deformations reach the load value of $P_{pl,full} = 1440$ kN, which is the theoretical plastic collapse load of a perfect plate (i.e. $\delta_0 = 0$) that is fully loaded along the edges (i.e. $s_s = 900$ mm). It should be noted though that for the shorter loaded lengths, the strains required in order to reach $P_{pl,full}$ are beyond those compatible with the small strain/displacement assumption of an MNA. Also, in all the cases, the corresponding collapse mechanisms feature yielding of excessive regions of the finite element models. Since (i) these types of mechanisms are not representative of the actual collapse mechanisms, (ii) the load-deformation paths of the plates with smaller loaded lengths still continue to rise with unrealistic values of deformations violating the small displacement/strain assumption of the MNA and (iii) reaching the upper-bound plastic collapse load value of the fully-loaded perfect plate (i.e. $P_{pl,full} = 1440$ kN) regardless of the loaded length is nonsensical, a clear need for criteria to determine realistic estimates of plastic collapse loads from numerical analyses is highlighted. Similar observations were made by [4]. Addressing this point is the focus of the present paper. An

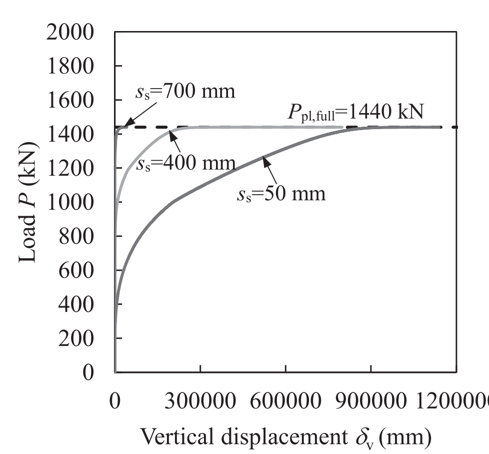

Fig. 2. Load-deformation paths for steel plates with different loaded lengths s_s.