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The linear analysis of thin shell problems using the numerical manifold method

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ABSTRACT

Although the numerical manifold method (NMM) has successfully solved many solid and flow problems, it has scarcely touched upon the analysis of shell problems. A shell is a genuine visible manifold in the real world and in principle NMM is pretty suitable to solve shell problems. This study aims to fill in the gap by employing the Naghdi shell model to establish NMM for shell analysis. The mathematical cover is constructed using the lines of curvature on the shell middle surface so that the best interpolation precision can be ensured. Thereafter, the standard 4-noded isoparametric shape functions are taken as the weight functions subordinate to the mathematical cover. Taking advantage of the favorable features of NMM, especially the facility to perform the p-adaptive analysis without compromising the element conformity, a high-order NMM model is employed to suppress the membrane and shear locking phenomena. Several typical shell benchmarks have been analyzed to testify the performance of the formulation in the static analysis of thin shell problems, manifesting that the proposed method can yield desirable results for linear thin shell problems, thus richly deserving the word "manifold".

1. Introduction

Shell structures are ubiquitously employed in engineering, attributing to its congenial properties, in a way, especially the efficiency of load-carrying behaviors, high stiffness and aesthetic value. For the past decades, the finite element method (FEM) has been extensively applied in the analysis of shell problems, with mainly three distinct approaches, namely (i) flat faceted elements, (ii) curved elements and (iii) degenerated shell elements [1,2]. Impressive as it is, the shell FEM is believed to have the following disadvantages: 1) the precision of stresses which are derived from differentiating displacements is obviously lower than that of displacements; 2) a high-quality mesh is invariably required [3]; 3) the geometry discontinuity caused by the discretization of the shell surface with flat elements occurs, 4) the severe repercussions of membrane and shear locking, in which the locking of pure displacement-based shell schemes cannot be wiped out [4–6]. Aiming at coping with those issues of shell FEM formulations, various techniques have been proposed as follows.

Initially some stress post-processing technologies have been developed, including the smoothing technology suggested in [7–9], which improved the stress accuracy. [10–12] have been put forward to mitigate mesh-dependence and to improve efficiency, accuracy and stability of shell elements even in case that meshes are coarsely structured or elements are poorly-shaped.

Additionally, a number of avenues have been paved to attenuate the locking phenomenon, including the reduced integration or selective reduced integration [13–16], the Assumed Natural Strain (ANS) method [17,18], the Enhanced Assumed Strain (EAS) method [19–21] and the Discrete Shear Gap (DSG) method [22–24]. Conducive as they are, the selective or reduced integration schemes may create ill-conditioned system matrices due to the rank deficiency and spurious zero-energy modes [3]. Regarding the ANS method, Bathe et al. [3,10,25] generalized the ANS plate elements to degenerated shell elements invariably referred as MITC (Mixed Interpolation of Tensorial Components) elements, which has extensively been utilized in commercial FEM analysis software. The EAS method involves the utilization of the three field variational principle of Hu-Washizu, and is applied in shell structural analysis in both the linear and non-linear problems [20,26]. The DSG method shares some common points with the ANS method, namely, the shear strain is modified within the element. Yet, DSG does not deploy collocation points or introduction of extra degrees of freedom and is able to work for arbitrary polynomials and element shapes [22].

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Meanwhile, various newly created numerical methods have been exerted to surmount those obstacles in the shell FEM analysis. Meshless methods were inaugurated by Monaghan et al. [27] and Lucy [28] in the modeling of astrophysics phenomena, which was then named Smoothed Particle Hydrodynamics (SPH). And it was further developed and applied in the shell analysis by many researchers [29–31]. The partition of unity method (PU) was incipiently proposed by Babuska [32], which enables the establishment of higher-order global approximations without adding external nodes [33]. A higher accuracy and convergence rate are thus obtained. Then a series of PU based generations and extensions to FEM were formulated to overcome those disadvantages in FEM analysis, which made striking strides in the application of shell analysis as well. The common feature of these methods is that a partition of unity is utilized to establish the global approximation by "pasting together" the local approximation. In particular, smooth, discontinuous, singular or numerical enrichment functions are comprised in local approximations in order to accommodate local characteristics of solution. For instance, the generalized finite element method (GFEM) [34–36], which eliminates locking phenomena and yields desirable numerical results. Moreover, The Smoothed Finite Element Method (SFEM) was ushered in by Liu [37] in 2006. Nguyen-Van, Wang et al. [38–41] then applied SFEM in the analysis of plates and shells, which smoothed the strain tensors, avoided the transverse shear locking and elevated the accuracy of coarse meshed schemes. Additionally, Bathe illustrated the finite element method enriched by interpolation covers in the CST2014 conference [42], and then utilized the interpolation covers for the MITC3, MIT4 shell elements [43,44], which, on the one hand, enhanced convergence rate solution accuracy without local mesh refinements; on the other hand, improved membrane and bending behaviors.

Meanwhile, Isogeometric analysis, proposed by Hughes et al. [45,46], which combines CAD and computer aided engineering by adopting the same basis function for both geometric description and unknown field variables, has been widely applied in the modeling of thin shells. And, the C^1-continuity of Kirchhoff-Love shell model can be satisfied by IGA discretization. For example, Kiendl et al. studied both Reissner-Mindlin (5-parameter) [47,48] and Kirchhoff-Love (3-parameter) [49–52] shell models with isogeometric analysis. The bending strip method was used in imposing the C^1-continuity of Kirchhoff-Love shell structures with multiple patches in [50]. Duong et al. presented a general non-linear computational formulation for rotation-free thin shell models with isogeometric analysis [53]. Sauer et al. studied the rotation-free three parameters shell model, which proved to be LBB stable [54]. Nguyen-Thanh et al. [55] brought up the RHT-splines for multiple-patch coupling for large deformation of thin shells with isogeometric analysis.

It was in 1991 that Shi proposed the numerical manifold method (NMM), aiming at abridging the chasm between the continuum (e.g. FEM) and dis-continuum methods (e.g. DDA, acronym for Discontinuous Deformation Analysis) [56]. Two covers, namely the mathematical cover and the physical cover, are uniquely introduced in NMM, which enables an agreeable consistency with both FEM and DDA. Hence, continuous, jointed, blocky materials can be computed in a consistent manner. Recent years have witnessed a buoyant application of NMM in multiple fields [57–64].

A major advantage of NMM over FEM is that the p-version adaptive analysis can be executed without any shackles. The higher degree polynomials were utilized as the local approximations by Chen et al. [65] and Jiang et al. [66] to drive the higher-order NMM schemes for second order problems, which, however, suffered the rank deficiency of the global stiffness matrix. An et al. [67] proposed an algorithm based on the inherent topological information of the finite element cover for predicting the rank deficiency of the stiffness matrix, and extended in [68]. A variety of strategies targeting on eliminating the rank deficiency were suggested by [69–71].

The applications of NMM in solid mechanics have been burgeoning. Its implementation in the plate and shell analysis are primarily concerned here. Zhou et al. [72] applied NMM based on the constrained variational principle to analyze beams and plates. The introduction of the penalty function to the generalized variational principle is suited to impose constraints implicitly in the function or to meet the required inter-element continuity. Wen and Jian worked on the C^1 continuous cubic B-spline surface interpolation and devised the B-spline based NMM, which enhanced the solution accuracy and convergence rate [73]. The polygonal manifold element was also put forward by Wen, which is favorably adjusted to the intricate computational domain [74]. Zheng et al. [75] formulated the numerical manifold method of Hermitian form and applied it in solving the fourth-order problems regarding Kirchhoff thin plate bending. Those applications significantly boost the development of NMM. Remarkable as it is, scarcely has NMM been implemented in analyzing shell problems. As a shell is a genuine visible manifold in the real world, only a successful application of NMM to shell problems NMM is worthy of the word "manifold" in NMM. This study just aims to fill in the gap.

This paper is organized as follows: in Section 2 we give a brief introduction to the fundamentals of shell formulations, from the Naghdi shell model, geometry preliminaries to the kinematics, which lays the foundation for the derivation of shell NMM. Section 3 gives discrete equations of NMM in analyzing shell problems. The NMM for the shell configuration, from the establishment of cover systems, the weight functions, the local approximations to its variational forms, are presented in this section. Finally, several testing benchmarks are studied in Section 4, which are compared with the reference solutions. Section 5 emphasizes on the conclusions.

2. Fundamentals concerning shell structures

The mathematical models with different physical assumptions to describe shell structures can be boiled down into mainly two categories [1]. The Koiter model is based on the Kirchhoff–Love hypothesis, which falls into the fourth-order problems. According to the primal variational formulation, the weight functions in the PU requires to be globally C^1 continuous but piecewise C^2 continuous, or more accurately, H^2 regular, which is not convenient numerically. Whereas the Naghdi model [76] is based on the Reissner-Mindlin assumptions that take into consideration the transverse shear deformation [77], falling into the second-order problems. The weight functions in the PU, on the other hand, needs to be merely globally C^0 continuous but piecewise C^1 continuous, or more accurately, H^1 regular. The utilization of the Naghdi shell model is, therefore, of computationally advantageous. Therefore, the Naghdi shell model is more extensively adopted in the literature.

2.1. Geometrical preliminary

The geometry of a Naghdi shell is described by the middle surface, denoted by S, as is delineated in Fig. 1. The orthogonal curvilinear coordinate system (α, β, γ) is established on the middle surface in the establishment of Naghdi shell model.

Various geometrical parameters, regarding the kinematics in the subsequent section, are expatiated here:

Principal curvatures of the middle surface S are denoted by (k_1, k_2). A, B are defined as the Lamé coefficients, in terms of which, the Lamé coefficients of any equidistant surfaces from S are expressed as [1],

$$\begin{aligned} H_1 &= A(1+k_1\gamma), \\ H_2 &= B(1+k_2\gamma), \\ H_3 &= 1. \end{aligned} \tag{1}$$

with $\gamma \in [-\frac{t}{2}, \frac{t}{2}]$, and $t =$ thickness of the shell.

The differential elemental area in the system of curvilinear coordinates of S is given by