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Development of a viscoelastic spline finite strip formulation for transient analysis of plates

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A B S T R A C T

In this study, the forced vibration behavior of viscoelastic plates is investigated using the spline finite strip method (spline FSM). It is assumed that the viscoelastic behavior of materials considered in this research is linear, which means the material constants are independent of load level and are a function of time only. The shear modulus is thus represented as a function of the so-called Prony series. Moreover, it is assumed that the bulk modulus remains constant. For spline FSM, the equilibrium equations are derived using the principle of virtual work and the classical bending plate theory in matrix form. A numerical solution is developed for forced vibration analyses using the Newmark's method. The damping effects corresponding to the viscoelastic nature of materials are incorporated in the analysis by a novel approach that defines a fictitious damping force vector. The convergence of the spline finite strip method for a viscoelastic plate under different loading conditions is studied. Additionally, the fast fourier transform (FFT) of forced vibration responses is applied to evaluate the natural frequencies of plates. The accuracy and efficiency of the proposed spline finite strip model are verified by comparing the simulation outcomes presented in this work with the results obtained from the finite element method using ANSYS. Through a comparison of the results, we found the spline finite strip results to be in good agreement with the finite element results.

1. Introduction

Thin-walled structures are ubiquitous in many branches of modern engineering, with areas of applications ranging from aircrafts, ships and space vehicles to bridges, buildings and storage vessels. Thin-walled structures made of polymers and reinforced polymer composites are also utilized for constructing large lightweight structures. Indeed, these polymers are well-known as a viscoelastic material that may give inaccurate results by linear elastic analysis. Thus, a major challenge in designing polymer thin-walled structures is their time-dependent behavior originating from material viscoelasticity properties.

The finite element method (FEM) is a powerful and versatile tool for structural analysis and is used to perform precise analysis on structures. The finite strip method (FSM) is a version of FEM that utilizes a special element called a "strip". The distinctive features of finite strip analysis are computational economy and ease in structural modeling. This method was classified in two versions, namely semi-analytical FSM and spline FSM. The differences between the various versions of FSM can be found in their choice of estimating functions.

Spline FSM was first developed by Cheung in 1982 as an alternative to semi-analytical FSM [1]. Cheung *et al.* [2] later extended this method to include the free vibration and static analysis of arbitrary-shaped plates. These analyses clearly demonstrated the versatility of the method. Dawe and Wang [3] presented the vibration of shear-deformable beams using the B-spline function that eliminated the effects of shear-locking, while Sheikh and Mukhopadhyay [4] studied the linear and nonlinear transient vibrations of plates with arbitrary shapes and stiffeners, and with arbitrary orientation by the spline FSM. Fazilati and Ovesy [5] developed a semi-analytical as well as a spline finite strip method for the analysis of the dynamic instability behavior of flat and curved thin-walled composite laminated structures under harmonic axial in-plane loads. In other studies, Ovesy and Fazilati [6] formulated and applied two different versions of FSM (spline and semi-analytical FSM) to analyze the problem of parametric instability of laminated curved panels under non-uniform end loading. Assaee and Hasani [7] developed a spline finite strip approach for investigating the forced vibration behavior of thin-walled composite circular cylindrical shells. Various transient loads as well as different boundary conditions were analyzed.

The literature on the transient vibration of viscoelastic material is rather limited. Damping is the most important parameter to consider when studying forced vibration analysis. Although finite element

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analysis has become a popular tool for modeling structures, the main challenge in using an FE model is predicting damping in structures, especially when viscoelastic materials are incorporated into a structure for additional damping.

Dalenbering [8] investigated the force vibration response of two different composite double-layer plate structures based on traditional linear viscoelasticity, using FE models. Hammrannd [9] studied the dynamic analysis of linear viscoelastic beams, rings, plates and shells with regards to the time-dependent effects of viscoelastic material using the FEM. Babouskos and Katsikadelis [10] investigated the nonlinear dynamic response of thin plates made of linear viscoelastic material of the fractional derivative type, using the Analog Equation Method (AEM). Ghayesh [11] analytically investigated the free and forced vibrations of a Kelvin-Voigt viscoelastic beam supported by a nonlinear spring, using the multiple timescales method. Ghayesh [11] also characterized the effect of system parameters on linear and nonlinear natural frequencies and frequency-responses. Kiasat *et al.* [12] studied the free vibration of isotropic viscoelastic beams and plates on a viscoelastic medium, based on Boltzmann's superposition integral model, using Dynamic Mechanical Analysis (DMA). Zhang *et al.* [13] developed a Fourier expansion-based differential quadrature (FDQ) method to analyze numerically the transverse nonlinear vibrations of an axially accelerating viscoelastic beam. Amabili [14] studied the nonlinear vibration behavior of Kelvin–Voigt viscoelastic thin rectangular plates subjected to normal harmonic excitation in the spectral neighborhood of the lowest resonances. Comparison to viscous damping, effect of neglecting nonlinear viscoelastic damping terms, change of the frequency–response with the retardation time parameter and the effect of geometric imperfections were studied.

The forced vibration analysis of viscoelastic materials using Finite Strip Method has not yet been presented in the literature. This task is fulfilled in this paper by performing an investigation on the force vibration behavior of thin plates made of viscoelastic material. To check the validity of the developed methods, the results obtained by finite element analysis using ANSYS was used for comparison purposes.

In this research, viscoelastic plates are subjected to different loading and boundary conditions, and the effects of damping on time-history responses due to viscoelasticity effects are studied. As well, the natural frequencies of plates made of viscoelastic materials are obtained from the time-history responses of plates by implementing fast Fourier transform (FFT).

2. Theoretical developments

2.1. Linear viscoelastic material models

Viscoelastic material behavior has been represented through various rheological models. One of the models which has received a great deal of attention by researchers is the Prony series. In this model, the shear modulus is expressed as a function of time, as follows:

$$G(t) = G_\infty + \sum_{i=1}^{n} G_i \exp\left(-\frac{t}{\tau_i}\right) \tag{1}$$

In the above equation, $G(t)$ is the time-dependent shear modulus, G_∞ is the asymptotic value of shear modulus, G_i coefficients are constant values, and τ_i are the relaxation times. All of the constants are derived from experimental tests for a viscoelastic material. Much of the research has assumed that the bulk modulus of viscoelastic material remains constant during a rheological process [15]. Using this assumption and Eq. (1), the relations for evaluating the time-dependent elastic modulus and Poisson's ratio for the viscoelastic materials is revealed.

The response of viscoelastic materials is associated with a history of loading. For a one-dimensional loading case, the constitutive equation relating the history of stress and strain during loading for a linear viscoelastic material is expressed by means of a hereditary integral, as in the following:

$$\boldsymbol{\sigma}(t) = \varepsilon(t)E(0) - \int_{0^+}^{t} \frac{dE(t-\tau)}{d\tau}\varepsilon(\tau)d\tau \tag{2}$$

where $\boldsymbol{\sigma}(t)$ and $\varepsilon(t)$ are time-dependent stress and strain tensors, respectively, and $E(t)$ is the time-dependent elastic modulus for a viscoelastic material. It should be noted that only linear viscoelastic material properties are considered in this paper. This means that the material constants are independent of load level and are a function of time only.

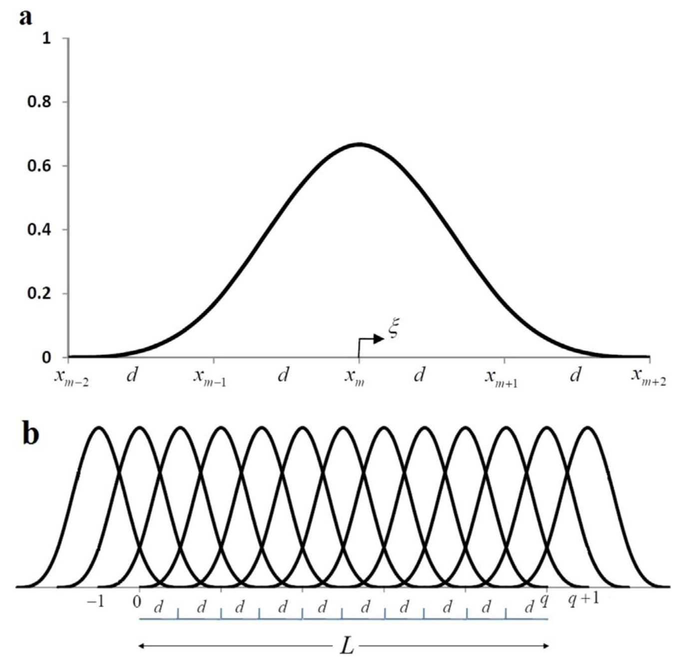

Fig. 1. (a) Local cubic spline function, (b) B3 Spline series.

2.2. Introduction to spline FSM formulation

The spline function is an estimating continuous-piecewise defined function. It is represented in the form of B1- B2- B3- B4-spline and B5-spline. The B3-spline function, which is presented as Eq. (3), is used in the current study. This function is depicted in Fig. 1a.

$$\varphi_m = \frac{1}{6}\begin{cases} 0 & \xi \le -2 \\ (\xi+2)^3 & -2 \le \xi \le -1 \\ (\xi+2)^3 - 4(\xi+1)^3 & -1 \le \xi \le 0 \\ (2-\xi)^3 - 4(1-\xi)^3 & 0 \le \xi \le 1 \\ (2-\xi)^3 & 1 \le \xi \le 2 \\ 0 & \xi \ge 2 \end{cases} \qquad \xi = \frac{x-x_m}{d} \tag{3}$$

In the above equation, x_m is the coordinate of the center of the local spline function. To interpolate the general function $f(x)$ in the interval of $a < x < b$ using the spline function, this interval is divided into equal pieces, where the distance between any two nodes is $d = (a-b)/q$. As a result, a total of $q+1$ nodes will be obtained. Corresponding to each node, a spline function is assumed and the spline series is finally obtained, as shown in Fig. 1b. In order to apply essential boundary conditions in the B3-spline function, two nodes are added to the beginning and end of the above-mentioned domain.

Therefore, a representation of an arbitrary function $f(x)$ using the spline series may be defined as Eq. (4):