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Rapid identification of pre-buckling states: A case of cylindrical shell

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ABSTRACT

The problem to identify pre-buckling states for thin-walled shell corresponds to the problem to identify pre-bifurcation solutions (the inverse bifurcation problem) for von Karman equations that govern the structure. Typical solution sequences similar to those of post-bifurcation solutions observed along the bifurcation paths of the nonlinear boundary problem for von Karman equations are extracted to serve as precursors of bifurcation (tools to solve the problem). The method allows one to divide all operations required to solve the problem under study into two non-equal parts. The most time-consuming part (to trace bifurcation paths and cluster the respective solution) is performed off-line, while the part of the algorithm that is carried out on-line (the identification algorithm) requires a relatively small number of arithmetic operations. This allows development of the efficient system of rapid identification of pre-buckling states.

1. Introduction

The bifurcation theory employed to investigate a thin-shell structure makes it possible to consider both direct and inverse bifurcation problems. The direct bifurcation problem, the most conventional one, implies that one estimates buckling (bifurcation) loads for various external loadings, boundary conditions and so on. As far as dependences of buckling loads on problem parameters are strongly non-monotonous it is necessary (in order to solve the problem) to trace all its bifurcation paths and ascertain its complete bifurcation set (for example, [1]).

The term 'inverse bifurcation problem' is conventionally used in two distinct senses. The first statement suggests that one seeks for such values of problem parameters that the respective buckling load satisfies certain demands – by way of illustration we may point to the problems to find the worst initial imperfection or the infavourable load [2] (that is the imperfection/load corresponding to the lowest possible buckling load). Papers [3–5] deal with an approach (to solve this problem) based upon specific perturbation functions with single or multiple localized dents (dimple-shape imperfections); to find the worst imperfection, authors propose to find minimum buckling load among those corresponding the perturbations typical for experimental studies [3,5–7]; another approach employs nonlinear buckling modes as perturbations [5]; Schenk and Schuëller [8] propose to analyze statistically experimental post-buckling shapes [7].

In our view, this problem can be solved by tracing all its post-buckling (bifurcation) paths, since nonlinear buckling modes associated with the lowest buckling loads correspond to (secondary and tertiary) bifurcation paths with relatively small low boundaries of existence domains [1]. The deformed shapes corresponding to these paths, usually, a single dent, a group of dents, or a 'belt' of dents [1], are similar to those employed in single or multiple perturbation load approach [3,5,6].

The second statement of the inverse bifurcation problem implies that one attempts to predict buckling (or to put it differently, to identify pre-buckling state) provided a sequence of deformed shapes is observed. This statement is a subject of much current interest as far as it manifests itself in actual practice as the problem of rapid sustainability assessment of a damaged thin-walled structure. On the other hand, robust design [9], which is growing more popular in engineering, implies that one is able rapidly identify every possible buckling state.

The present paper concerns with a novel approach to predict thin-walled shell buckling that it is *the second statement* of the inverse bifurcation problem for thin-shell structures. The approach utilizes knowledge about post-buckling (bifurcation) paths traced for the respective static nonlinear elastic problems – namely, typical sequences of solutions (deformed shapes) associated with post-buckling bifurcation paths serve as bifurcation precursors; the observed sequences of deformed shapes may correspond to processes unfolding in time. It is worth stressing that in the frameworks of dynamical analysis for this type of partial differential equations (PDEs) it is possible to solve the inverse bifurcation problem for a particular right-hand member (load function) only; while the proposed approach can be employed to identify pre-buckling state for any right-hand member.

Rapid assessment is associated with the concept of progressive

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disproportionate collapse of a structure. For example, the paper [10] discusses progressive collapse for a structure made of box-like beams and possible mechanism of its occurrence. The paper [11] reviews various approaches to rapid assessment and describes the respective problems that have not yet been adequately explored. By and large all available approaches can be broadly classified into two groups. For the method of the first group, one identifies values of parameters of the observed system (those of initial imperfection, the external load, flaws or irregularities of shell material) and then performs non-linear buckling analysis for the system with the identified parameters to estimate buckling load (see [12] and references therein). The second group suggests that one utilizes a salient pre-bifurcation feature to predict buckling; such features are collectively called 'flags', 'fingerprints', or 'precursors' of bifurcation; recently, terms 'early-warning signs' and 'tipping points' are growing more popular [13–15]. Sometimes methods that belong to these groups are named model and modelless, respectively [16].

The method proposed by Stull et al. [17] exemplifies the first approach as applied to theory of thin-walled structures; it implies that the stochastic inverse problem is solved (in the framework of Bayesian statistics) to identify initial imperfections and thereby to estimate the buckling load. Vibration Correlation Technique, experimental method proposed by Abramovich et al., exemplifies the second approach [18]; the authors propose to use characteristic oscillations preceding to buckling as precursors of bifurcation and formulate operational guidelines to prevent buckling.

It is worth emphasizing that precursors of bifurcation for PDEs and particularly for equations of thin-walled shells theory fall far short of being perfect, "Despite these exploratory works, it is quite clear at this point that the full mathematical analysis of early-warning signs for spatio-temporal systems is largely uncharted territory."[13] A plenty of precursors designed for single-dimension time series can be extended to multi-dimension ones in a straightforward fashion and therefore used as those for PDE: it is possible to use precursors based on rate of change of covariance matrix, skewness, and so on [19]. Another approach leads to averaging with respect to spatial coordinates that makes it possible to employ the bifurcation precursors designed for single-dimension time series as such [20].

For the overwhelming majority of methods used to identify a pre-buckling state, it is necessary to perform a very large number of arithmetic operations. Since speed of buckling processes associated with thin-walled shells is very high and the buckling can be due to off-design contingencies, the use of such methods for rapid sustainability assessment of thin-walled shells is computationally prohibitive.

The present paper deals with the method able to divide all operations required to solve the problem under study into two non-equal parts. The substantially greater part can be performed off-line; the significantly lesser – directly used to identify pre-buckling states – is to be carried out on-line. The necessary precondition to apply the method is to trace all bifurcation path of the respective non-linear boundary problem.

The seminal papers concerned with bifurcation paths of von Karman equations (for example, [21]) deal mainly with the primary bifurcation paths. However, the non-linear boundary problem features the spectrum crowding and the secondary and tertiary bifurcation paths that makes it necessary to use a radically new method. To the best of authors' knowledge, the paper [22] presented the secondary bifurcation paths associated with a cylindrical shell subjected to uniform external pressure; the solutions of the secondary paths correspond to localized buckling shapes, frequently encountered in actual practice (see also [1,23]). The paper [24] considers the nonlinear boundary problem under investigation for cylindrical panel subjected to the lumped force; the authors employ the finite element method combined with arc-length technique to trace primary bifurcation paths.

The papers [25,26] are concerned with 'jump' technique to switch to bifurcation paths. The authors succeeded in tracing some secondary bifurcation paths for an axially-compressed cylindrical shell (the compression is uniform). Hu and Burgueño [27] explore, both numerically and experimentally, bifurcation solutions corresponding to non-uniform compression; the compression function is a linear combination of linear buckling modes. Zhao et al. [28] combine linear and nonlinear techniques to prevent localized buckling (associated with tertiary bifurcation paths [23]) by means of optimally placed grids-stiffeners. The monograph [23] presents the bifurcation structure including various primary (for cylindrical panel [see also [29]]); primary and secondary (for cylindrical shell subjected to a uniform pressure); primary, secondary, and tertiary (for cylindrical shell subjected to a uniform axial compression) bifurcation paths; papers [30,31] provides results for spherical shell.

One should emphasize that usual nonlinear computation can be carried out using any conventional finite elements package, while the problem to construct bifurcation structure makes it necessary to develop and implement specific methods. These methods may lean upon the finite element method [2,24] or may require development new methods to solve nonlinear boundary problems for PDEs [1,32] – we favour the second alternative as it avoids lots of computational difficulties of the bifurcation theory.

To summarize, the present paper proposes a novel approach to predict buckling for thin-shell structure (to identify pre-buckling states) – mathematically, it is the approach to solve the inverse bifurcation problem for von Karman equations. The remainder of the paper is organized as follows. The next section presents the von Karman equations and discusses briefly their post-bifurcation solutions. The third and fourth sections formally state the problem to identify pre-buckling state and outline the method used to solve it, respectively. The fifth one provides the identification results. Finally, the last section presents conclusions.

2. Nonlinear boundary problem of thin-walled structures theory

In the framework of shallow shells theory, a cylindrical shell subjected to a uniform external pressure is governed by von Karman equations for both pre- and post-buckling states [33]:

$$a_1\nabla^4 w + T(w,\Phi) - \nabla_k^2\Phi = q,$$
$$a_2\nabla^4\Phi - \frac{1}{2}T(w,w) - \nabla_k^2 w = 0, \tag{1}$$

where
$\nabla_k^2\alpha = k_1\frac{\partial^2\alpha}{\partial x_1^2} + k_2\frac{\partial^2\alpha}{\partial x_2^2}$;
$T(\alpha,\beta) \equiv \frac{\partial^2\alpha}{\partial x_1^2}\frac{\partial^2\beta}{\partial x_2^2} - 2\frac{\partial^2\alpha}{\partial x_1\partial x_2}\frac{\partial^2\beta}{\partial x_1\partial x_2} + \frac{\partial^2\alpha}{\partial x_2^2}\frac{\partial^2\beta}{\partial x_1^2}$;
$w = w(x_1, x_2)$, $\Phi = \Phi(x_1, x_2)$ are the normal displacements (of the shell middle surface) and the force function, respectively; $q = q(x_1, x_2)$ is a function of an external pressure. The problem is defined on a cylindrical domain $\Omega = \{0 \le x_2 \le 2\pi;\ 0 \le x_1 \le L/R\}$; $\Gamma \equiv \partial\Omega$ stands for its boundary. L, R, and h are the length, radius and thickness of the shell. E and μ denote the Young's modulus and Poisson's ratio of shell material, respectively.

The shell ends are simply supported: Eq. (1) is completed by the boundary conditions

$$w|_\Gamma = 0;\quad \left[\frac{\partial^2 w}{\partial x_1^2} + \mu\frac{\partial^2 w}{\partial x_2^2}\right]|_\Gamma = 0;\quad \Phi|_\Gamma = 0;\quad \frac{\partial\Phi}{\partial x_2}|_\Gamma = 0. \tag{2}$$

One should emphasize that the method discussed below is valid for various boundary conditions and thin-walled structures.

The figures presented in the paper are plotted for values $\Omega = \{0 \le x_1 \le 4;\ 0 \le x_2 \le 2\pi\}$, $a_1 = 0.1$; $a_2 = -1$; $k_2 = 0$; $\mu = 0.3$; these values correspond to the ratio of shell length to its radius equal to $L/R = 4.0$. The ratio of the radius to shell thickness $k_1 = R/h$ (with the respective wavenumber (in the circumferential direction) of the linear buckling mode corresponding to the minimal buckling pressure, the