Thin-Walled Structures 123 (2018) 11–24

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

GBT buckling analysis of generally loaded thin-walled members with arbitrary flat-walled cross-sections

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ARTICLE INFO

Keywords:
Thin-walled members
Elastic linear buckling analysis
Complex loadings
Generalised Beam Theory (GBT)
Arbitrary flat-walled cross-section members

ABSTRACT

This paper deals with extending the domain of applicability of a recently developed Generalised Beam Theory (GBT) formulation intended to perform elastic linear buckling analyses of thin-walled members (i) exhibiting arbitrary flat-walled cross-sections (including those combining closed cells and open branches), and (ii) acted by general loadings. These loadings, which include transverse forces acting away from the member shear centre axis, are termed "general" in the sense that they may involve the presence of pre-buckling stress distributions associated with any possible combination of all the stress tensor membrane components (σ_{xx}, σ_{ss} and τ_{xs}, for a plane stress state), including cell shear flows – therefore, all the relevant geometrically non-linear effects need to be taken into consideration. After briefly presenting the main concepts and procedures involved in the development and implementation of the above GBT formulation, this same formulation is employed to analyse the buckling behaviour of beams with different types of cross-section geometry (containing closed cells) and exhibiting different loading and support conditions. In particular, they consist of (i) a RHS cantilever acted by two tip point loads, (ii) a closed-flange I-section simply supported beam subjected to a uniformly distributed load and (iii) a two-cell RHS section cantilever acted by tip transverse forces and couples. In all cases, the loads are applied both along the shear centre axis and also along axes parallel to it and located at the beam top and bottom surfaces. The results presented and discussed, which consist of pre-buckling stress fields, buckling curves and buckling mode shapes, are obtained by means of the newly released code GBTUL 2.0 and validated by means of the comparison with shell finite element values obtained with the code ANSYS.

1. Introduction

Evaluating the structural efficiency of thin-walled steel members usually requires assessing their linear buckling behaviour, a task that involves determining the relevant buckling modes and the associated bifurcation loads/stresses. However, the difficulty in performing this task tends to increase with (i) the cross-section shape complexity and (ii) the slenderness of its walls, which is responsible for the appearance of buckling modes combining local, distortional, shear and wall transverse extension deformation patterns. Moreover, in members with cross-sections containing closed cells, it is also well known that torsion causes non-null shear strains associated with the presence of cell shear flows (*e.g.*, [1]).

Generalised Beam Theory (GBT) is a thin-walled bar theory that is able to capture all the deformation patterns mentioned in the previous paragraph, thus providing a very competitive alternative to the traditional shell finite element method (SFEM) or the equally novel constrained finite strip method (cFSM). The main distinctive feature and advantage of this technique is the fact that it expresses the cross-section deformation as a linear combination of contributions from structurally meaningful *cross-section deformation modes*. Each "contribution" consists of a product involving (i) a shape function defined along the cross-section mid-line (the deformation mode profile) and (ii) the corresponding longitudinal modal amplitude function, providing the variation of the deformation mode amplitude along the member length. The sets of procedures leading to the determination of the each of the above shape and amplitude functions are termed *cross-section analysis* and *member analysis*, respectively.

Originally proposed by Schardt [2,3], GBT has been considerably developed in the last 15 years, mostly due to the research effort carried out at the University of Lisbon (*e.g.*, [4–7]), where it has been employed in the context of a wide variety of structural problems, involving

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https://doi.org/10.1016/j.tws.2017.10.045
Received 17 August 2017; Received in revised form 26 October 2017; Accepted 31 October 2017

members and structural systems (frames or trusses). Linear buckling analysis has always been a major area of research in the context of GBT [7] – the more recent works have addressed various topics, such as convex polygonal regular tubes [8–10], composite members [11,12], web crippling [13,14], cold-formed steel structural systems [15–17], GBT-based exact finite elements [18] and general loading conditions [19]. Other topics of the areas of GBT research at the University of Lisbon include post-buckling (*e.g.*, [20–22]) and vibration/dynamic (*e.g.*, [23–25]) analyses.

However, it should not be inferred from the above list of references that the research activity on GBT has been taking place only at the University of Lisbon. In fact, GBT is currently a topic of research worldwide, as attested by the growing numbers of publications originating from a plethora of countries. In this context, a few recent works, in which GBT-based buckling analysis techniques are employed and/or developed, are mentioned next. Miranda et al. [26], for example, have developed a geometrically non-linear co-rotational formulation of GBT and used it to study the local and distortional buckling behaviour of members with complex (but open and unbranched) cross-sections. Taig et al. [27] presented an original cross-section analysis and an analytical approach to perform pre-buckling and buckling analyses of thin-walled members, which was applied to analyse lipped channels under combined axial and transverse loads. The application of GBT in the context of the buckling analysis of perforated cold-formed members has been addressed (i) by Casafont et al. [28], who used a heterogeneous GBT-based finite element mesh (with different properties for the perforated and non-perforated segments), and (ii) by Nedelcu [29], who proposed a technique that "translates" a buckling mode provided by a shell finite element model into the GBT modal language. Very recently, this last author also proposed a GBT-based generalisation of the Ayrton-Perry formula [30].

One work that deserves to be specially mentioned, due to the fact that it has significantly extended the domain of application of GBT buckling analysis, is due to Basaglia and Camotim [19]. It developed, numerically implemented and validated a formulation capable of handling arbitrary loadings, including transverse loads acting away from the member shear centre axis. However, this formulation – like the one reported in [27] – was applied exclusively in the context of open-section thin-walled members, *i.e.*, did not cover members with cross-sections containing closed cells. On the other hand, the authors have recently developed a novel cross-section analysis procedure capable of handling efficiently arbitrary flat-walled cross-sections [31,32].

The objective of this work is to combine the two aforementioned developments and assess the quality of the GBT buckling results in the context of members (namely beams) (i) exhibiting cross-sections with closed cells and (ii) subjected to general/arbitrary loadings. It is worth noting that the main specific mechanical feature of such members is the need to account for cell shear flow deformation patterns in the GBT buckling analyses – *e.g.*, the inclusion of one particular cell shear flow deformation mode is indispensable to capture adequately the member behaviour under torsion. The GBT-based results presented and discussed in this work are obtained by means of the code GBTUL 2.0, developed at the University of Lisbon and freely available online [33,34]. These results concern single and two-cell beams subjected to transverse loads acting on different locations with respect to the beam shear centre axis and causing pre-buckling states involving non-uniform bending and/or torsion – such states involve elastic pre-buckling stress distributions, obtained from preliminary first-order analyses, associated with any possible combination of all the stress tensor membrane components (σ_{xx}, σ_{ss} and τ_{xs}, for a plane stress state). For validation purposes, some of the GBT-based results are compared with values obtained through accurate shell finite element simulations performed in the commercial code ANSYS [35].

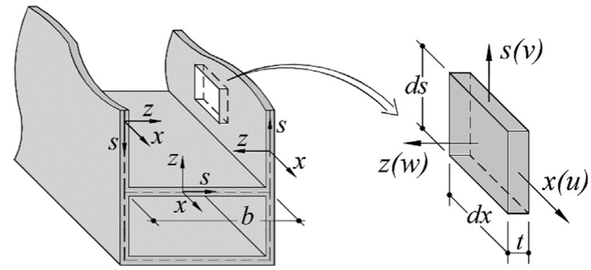

Fig. 1. Prismatic thin-walled member with a supposedly arbitrary flat-walled cross-section containing closed cells, and infinitesimal wall element with its local coordinate system and associated displacement components.

2. Generalised beam theory formulation – overview

As mentioned above, GBT is a one-dimensional bar theory that expresses/discretises the member deformed configuration as a linear combination of cross-section deformation modes multiplied by their (modal) amplitude functions. A very brief overview of GBT is presented next – more detailed accounts can be found in the literature (*e.g.*, [4–7,19]).

Consider the prismatic thin-walled member with the (supposedly arbitrary) flat-walled cross-section depicted in Fig. 1(a), in which local coordinate systems $x-s-z$ are adopted at each wall, as shown in Fig. 1(b). In GBT, the wall mid-plane axial, transverse and normal displacement components – $u(x,s)$, $v(x,s)$ and $w(x,s)$ – are given by (summation convention applies to subscript k)

$$u(x,s)=u_k(s)\varphi_{k,x}(x)\qquad v(x,s)=v_k(s)\varphi_k(x)\qquad w(x,s)=w_k(s)\varphi_k(x),\tag{1}$$

where (i) $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the mid-line functions defining cross-section deformation mode k (or "GBT mode k") (ii) $\varphi_k(x)$ or $\varphi_{k,x}(x)$ is the amplitude function describing its variation along the member length, and (iii) $1\leq k\leq N_d$, where N_d is the total number of deformation modes. Thus, the member deformed configuration can be expressed as a sum of contributions from the N_d deformation modes – the contribution of mode k is the product of its mid-line displacement functions by the corresponding amplitude function. Alternatively, (1) can be written in matrix form as

$$u=\boldsymbol{u}^T\boldsymbol{\varphi}_{,x}\qquad v=\boldsymbol{v}^T\boldsymbol{\varphi}\qquad w=\boldsymbol{w}^T\boldsymbol{\varphi},\tag{2}$$

where (i) $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are column vectors containing the $u_k(s)$, $v_k(s)$ and $w_k(s)$ functions, respectively, and (ii) $\boldsymbol{\varphi}$ is a column vector containing the corresponding amplitude functions $\varphi_k(x)$.

The member elastic strain energy U is given by (V is the member volume and the summation convention applies to subscripts i, j)

$$U=\frac{1}{2}\int_V\sigma_{ij}\varepsilon_{ij}dV,\tag{3}$$

where σ_{ij} and ε_{ij} are the various stress and strain components included in the analysis – after adopting the Kirchhoff-Love hypotheses, a plane stress state, with components σ_{xx}, σ_{ss} and τ_{xs}, takes place in the member mid-plane. Moreover, by using Eq. (2) and considering small strains and a St. Venant-Kirchhoff law, Eq. (3) can be expressed in the form

$$U=\frac{1}{2}\int_L(\boldsymbol{\varphi}_{,xx}^T\mathbf{C}\boldsymbol{\varphi}_{,xx}+\boldsymbol{\varphi}_{,x}^T\mathbf{D}\boldsymbol{\varphi}_{,x}+\boldsymbol{\varphi}^T\mathbf{B}\boldsymbol{\varphi}+\boldsymbol{\varphi}_{,xx}^T\mathbf{E}\boldsymbol{\varphi}+\boldsymbol{\varphi}^T\mathbf{E}^T\boldsymbol{\varphi}_{,xx})dx,\tag{4}$$

where L is the member length and $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{E}$ are $N_d\times N_d$ linear stiffness matrices, associated with various cross-section mechanical properties, namely (i) primary/secondary warping, (ii) transverse extension/flexure, (iii) plate distortion/torsion and (iv) membrane/flexural Poisson effects, respectively – the analytical expressions of their