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Parametric instability of a functionally graded cylindrical thin shell subjected to both axial disturbance and thermal environment

X. Li[a], C.C. Du[b], Y.H. Li[a,*]

[a] School of Mechanics & Engineering, Southwest Jiaotong University, Chengdu 610031, PR China
[b] School of Mechatronic Engineering, Southwest Petroleum University, Chengdu 610500, PR China

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ABSTRACT

This paper focuses on the parametric instability of a functionally graded (FG) cylindrical thin shell under both axial disturbance and thermal environment. Based on Love's thin shell theory, and considering the temperature-dependent properties of FG cylindrical shell, the dynamic equations of the FG cylindrical shell are derived by Hamilton's principle. The multiple scales method is performed to obtain the instability boundaries of the shell with axial disturbance. The primary and combination instabilities of the shell are studied systematically. Moreover, numerical simulations are utilized to discuss the influences of axial disturbed amplitude, material heterogeneity and thermal effects on instability regions, frequency characteristics of the shell. Specially, some numerical results are given to illustrate the combined influence of axial disturbed amplitude and temperature variation on instability regions.

1. Introduction

Functionally graded materials (FGMs) are advanced composite materials made of a mixture of ceramics and metals with smoothly and continuously varying material properties along one or more directions. In recent years, FGMs have attracted much interest in many research fields on account of their minimum weight, high strength and ultrahigh temperature resistance [1–14]. Specially, FGMs are generally accompanied with high temperature environment, which can lead to the change of mechanical properties of FGMs.

As a special type of shell structures, FG cylindrical shells have been wide spread in aerospace and mechanical engineering. Many researches conducted in the field of dynamic characteristics and vibration analysis of the FG cylindrical shells have been performed in the past several decades. Loy et al. [15] studied the vibration of FG cylindrical shells using Rayleigh-Ritz method based on Love's shell theory, they discussed the influence of constituent volume fractions and the constituent materials on the frequencies of the system. According to the higher order shear deformation theory, Matsunaga [16] investigated vibration and bulking of FG cylindrical shells using Hamilton's principle, and he obtained critical buckling stresses of simply supported FG cylindrical shells under axial loads. Alijani et al. [17] dealt with nonlinear vibrations of FG doubly curved shells using multi-modal energy approach. Numerical solutions of the shells under static and harmonic loads were obtained employing the pseudo-arclength continuation. Moreover, FG cylindrical shells are used primarily in high temperature environments. Malekzadeh et al. [18] studied free vibration of functionally graded truncated conical shells under thermal environment. Considering the two-dimensional axisymmetric temperature distribution, they obtained the initial thermal stresses by solving the thermo-elastic equilibrium equations. In Ref. [19], the coupled thermo-elastic and energy equations of FG cylindrical shells were established based on the second-order shear deformation shell theory. However, the changes of material properties generated by temperature distribution were not considered. Haddadpour et al. [20] researched free vibration of FG cylindrical shell subjected to thermal environment with different boundary conditions, and they defined the distribution of temperature based on steady state heat conduction in the thickness direction. Considering a primary resonance excitation and a 1:2 internal resonance, Du and Li [21] investigated the nonlinear vibration of a FG cylindrical shell subjected to thermal effects based on the multiple scale method. With the respect of thermal bucking of FG cylindrical shells, Shen [22,23] studied thermal postbuckling of a FG cylindrical thin shell with the consideration of the temperature-dependent properties of materials. A closed form solution of simply supported FG cylindrical shell with three types of thermal loading was presented by Wu et al. [24], and thermal bucking of the shell along longitudinal direction was also discussed in their paper.

Besides, it is worth mentioning that parameter vibration analysis of the FG cylindrical shells is also widely found in the papers. Yang and Shen [25] investigated the dynamic instability and free vibration of FG

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* Corresponding author at: School of Mechanics & Engineering, Southwest Jiaotong University, Chengdu 610031, PR China.
E-mail address: yhli2007@sina.com (Y.H. Li).

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cylindrical panels subjected to the forces under thermal environment. They employed a semi-analytical approach to obtain natural frequencies and instability regions of the panels. In Ref. [26], the parametric resonance of simply supported FG cylindrical shells subjected to pulsating in-plane loading was studied. The influence of the volume fraction of the material constituents on the unstable regions was systematically analyzed. For conventional cylindrical shell, Lam and Ng [27] analyzed parametric instability of the cylindrical shell under time-varying axial loads using four shells theories, and discussed the effects of length- and thickness-to-radius ratios on unstable regions. According to three-dimensional theory and Mathieu–Hill equations, parameter vibration analysis of piezoelectric circular cylindrical shells under the combination of axial loads and radial electric field was carried out by Zhu et al. [28]. They derived the instability regions of the shells using Bolotin's method. Furthermore, dynamic stability analysis of rotating cylindrical shell subjected to periodic axial loads was reported by Han et al. [29,30]. Based on Donnell's shell theory, the parametric resonance of rotating cylindrical shells subjected to periodic axial loading was studied in Ref. [31], the influence of the Coriolis forces on the unstable regions of the system was discussed particularly.

Moreover, the influence of axial disturbance on dynamic stability of FG cylindrical thin shells is significant. FG cylindrical thin shells subjected to axial disturbance are very common in the aeronautical and ocean engineering. Specifically, pulsation of engine thrust and sloshing of liquid fuel in the launch vehicle can lead to in-plane disturbance of the structures, which cause vibration and stability problems of the systems. Nevertheless, few researchers pay attention to the FG cylindrical thin shells under axial disturbance.

From an overall point of view, many significant results of FG cylindrical shells have been obtained in the literatures. However, to the best of authors' knowledge, combined influence of axial disturbance and temperature variation on dynamic stability of FG cylindrical thin shells have rarely been considered in papers. In the present work, parametric instability of a FG cylindrical thin shell under both axial disturbance and thermal environment is studied systematically considering temperature-dependent properties of materials. The effects of axial disturbance, thermal effects and material heterogeneity on instability regions, natural frequency of the shell are discussed.

2. Dynamic model

Fig. 1 shows a FG cylindrical thin shell with axial disturbance. The length and the thickness of the shell are denoted by L and h respectively. The radius of curvature of the middle surface is R. The shell is built in cylindrical coordinate system (x, θ, z). The displacements in the x, θ and z directions of the shell are expressed by symbols u, v and w, respectively. Besides, a disturbed displacement $S(t)$ along longitudinal x-axis is considered in Fig. 1.

Based on inhomogeneous properties of functionally graded materials through the gradient direction, neutral surface of FG cylindrical thin shells may not coincide with its geometric mid-surface. However, the differences between the results considering neutral surface and mid-surface are negligible according to Ref. [9,10] and authors' research (see Appendix A). In this paper, parametric instability of a FG cylindrical thin shell under both thermal effects and axial disturbance is based on geometric mid-surface of the shell.

Assuming that the FG cylindrical shell is constituted by a mixture of metal and ceramic components, and considering the temperature-dependent properties of the ones, we have [15]

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{1}$$

where coefficients P_i $(i = -1, 0, 1, 2, 3)$ are unique to the constituent

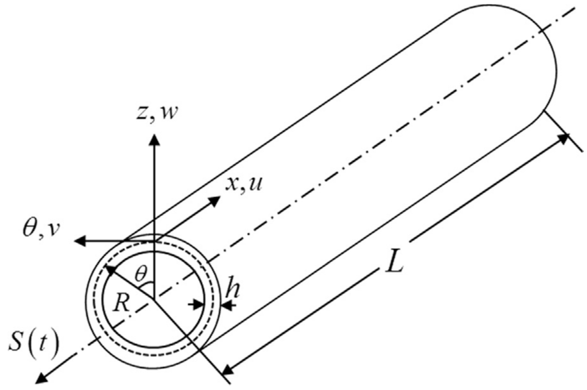

Fig. 1. FG cylindrical thin shell configuration with axial disturbance.

materials. $T = T^{\circ} + \Delta T$, in which T° is defined as initial temperature, and ΔT the uniform temperature rise. The material properties P (Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α and density ρ) are assumed to be graded along the thickness direction and can be expressed by a function of z-coordinate [15]

$$P(z, T) = [P_c(T) - P_m(T)]\left(\frac{2z + h}{2h}\right)^N + P_m(T) \tag{2}$$

where N is power law index, $0 \le N \le \infty$. The symbols c and m represent ceramic and metal respectively. It is found from Eq. (2) that the material of outer surface ($z = h/2$) is ceramic rich, and the inner surface ($z = -h/2$) is metal rich.

Based on Love's nonlinear thin shell theory [32], the strains are defined as functions of the normal coordinate z, namely

$$\varepsilon_x = \varepsilon_1 + z\kappa_1,\ \varepsilon_\theta = \varepsilon_2 + z\kappa_2,\ \varepsilon_{x\theta} = \gamma + z\chi \tag{3}$$

where ε_1, ε_2 and γ denote the middle surface strains, while κ_1, κ_2 and χ the middle surface curvatures. They are represented as

$$\varepsilon_1 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\ \varepsilon_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) + \frac{1}{2}\left(\frac{\partial w}{R\partial \theta}\right)^2,\ \gamma = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{R\partial \theta}, \tag{4}$$

$$\kappa_1 = -\frac{\partial^2 w}{\partial x^2},\ \kappa_2 = \frac{1}{R^2}\left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right),\ \chi = \frac{2}{R}\left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta}\right). \tag{5}$$

Considering the temperature variation, the constitutive relation of a FG cylindrical thin shell can be obtained

$$\boldsymbol{\sigma} = \mathbf{Q}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^T) \tag{6}$$

where $\boldsymbol{\sigma} = (\sigma_x\ \sigma_\theta\ \tau_{x\theta})^{\mathrm{t}}$ and $\boldsymbol{\varepsilon} = (\varepsilon_x\ \varepsilon_\theta\ \varepsilon_{x\theta})^{\mathrm{t}}$ are stress and strain vectors, respectively. $\boldsymbol{\varepsilon}^T = (\varepsilon_x^T \varepsilon_\theta^T 0)^t$ presents the thermal strain vectors. The letter t denotes the transposition of the vectors. $\mathbf{Q}$ is the material stiffness matrix [33].

Moreover, the thermal strains can be further expressed as

$$\begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_\theta^T \\ 0 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha(z, T) \\ \alpha(z, T) \end{bmatrix} \Delta T(z) \tag{7}$$

where $\alpha(z, T)$ denotes the thermal expansion coefficient.

Based on Mechanics of Composite Materials [33], force and moment resultants of the shell can be obtained