Thin-Walled Structures 124 (2018) 558–573

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

A buckling and postbuckling analysis of axially loaded thin-walled beams with point-symmetric open section using corotational finite element formulation

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ARTICLE INFO

Keywords:
Buckling
Postbuckling
Point symmetric section
Thin-walled beam
Corotational formulation
Finite element method

ABSTRACT

The axially loaded thin-walled beams with point symmetric open section are studied using a corotational finite element formulation. The kinematics of the beam element is described in the current element coordinate system. The element nodal forces are derived using the virtual work principle, and consistent second order linearization of the fully geometrically non-linear beam theory. Different axial load systems with the same centric resultant axial force but different resultant bimoments are considered. Numerical examples studied show that the effects of bimoments on the buckling and postbuckling behavior of point symmetric open section beams are remarked.

1. Introduction

Due to reduction of weight, material and cost, thin-walled beams with open section are extensively used in many structural applications: buildings, bridges, cars, ships, aircraft, etc. These types of structures can undergo large displacements and rotations without exceeding their elastic limits, and are designed to work under postbuckling conditions in some cases. Such flexible structures are susceptible to buckling. It is important to predict their global buckling loads and behavior up to the postbuckling range accurately. For accurate buckling loads, it is necessary to take into account the change in geometry induced by the prebuckling deformation, which is not considered in the linear stability analysis, and the buckling loads should be detected on the nonlinear load–deflection response of the structures.

Concentrically loaded thin-walled columns may globally buckle by (1) flexure about one of the principal axes (flectural buckling), (2) twisting about the shear center (torsional buckling), or (3) a combination of both flexure and twisting, called flexural–torsional buckling. Open sections that are doubly symmetric or point symmetric are not subject to flexural–torsional buckling because their shear center and centroid coincide.

It is well known that when a longitudinal force applied at an arbitrary point is transferred to the shear center, it will produce an additional bimoment proportional to the value of the warping function at this point [1]. Because the value of warping function ω_C at the centroid of the cross section is not zero for point symmetric Z section beam, a bimoment $P\omega_C$ is induced by the axial load P applied at the centroid of the Z section. The corresponding prebuckling deformations on the primary equilibrium path consist of axial deformations and twist deformations. Thus, with respect to the twist deformations on the primary path induced by the bimoments, the value of the warping function at the loading point may be regarded as an initial load eccentricity.

In [2], a twist simply supported Z section beam subjected to centric compression forces at both ends is studied. The bimoment induced by axial loads is considered as a part of a fundamental stress field, while the corresponding prebuckling twist deformations are assumed to be sufficiently small and are neglected. It is reported that the influence of the prebuckling bimoment on the pure torsional buckling load could be of importance for the axially compressed thin-walled Z section beams through the linear buckling analysis. However, the magnitudes of the prebuckling deformations are not checked in [2] to verify the validity of the assumption of small prebuckling deformations.

The global buckling analysis of axially loaded thin-walled beams has been extensively studied and is covered in various textbooks, e.g. [3–9]. However, to the authors' knowledge, the influence of bimoment and the prebuckling twist deformations on the global buckling and postbuckling behavior of axially loaded point symmetric thin-walled beams is not reported in the literature.

The objective of this paper is to investigate the influence of bimoment and the prebuckling twist deformations on the critical state, the

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https://doi.org/10.1016/j.tws.2017.11.056
Received 22 June 2017; Received in revised form 28 November 2017; Accepted 30 November 2017

failure mode of the global buckling, and the postbuckling behavior of axially loaded point symmetric thin-walled beams using a corotational finite element formulation. Different axial load systems with the same centric resultant axial force but different bimoments are considered in this paper.

In [10] a consistent corotational finite element formulation for the geometric nonlinear buckling and postbuckling analysis of thin-walled beams with generic open section is proposed. In order to include the nonlinear coupling among the bending, twisting, and stretching deformations, the second order terms of deformations parameters and their spatial derivatives, that will not converge to zero with the decrease of element size, and the third order term of the twist rate are retained in the element deformation nodal forces. This element was proven to be very effective for geometric nonlinear analysis of three dimensional beams with generic open section by numerical examples studied in [10]. However, because the centroid and the shear center are not coincident for thin-walled beams with generic open section, its kinematics is much more complicated than that of point symmetric beams. The expression of the element deformation nodal forces and the element tangent stiffness matrix are very complicated for thin-walled beam element with generic open section. The beam element proposed in [10] may be inefficient for the analysis of point symmetric thin-walled beams. The centroid and the shear center are coincident for point symmetric or doubly symmetric cross sections. Thus, the kinematic assumptions proposed in [11] for doubly symmetric beam element can be adapted and used here for point symmetric beam element. For thin-walled beams with point symmetric cross sections, it seems that the complexity of the element nodal forces and the element tangent stiffness matrix could be significantly reduced by considering the characteristics of section properties and the coincidence of the centroid and the shear center for point symmetric cross sections. Hence, the CR formulation proposed in [10] and the kinematics of the beam element proposed in [11] are adapted and used in this paper. The deformation nodal forces of the beam element are systematically derived by the virtual work principle and consistent linearization of the fully geometrically non-linear beam theory [10,11] in the element coordinates constructed in the current configuration using the procedure proposed in [12].

An incremental-iterative method based on the Newton-Raphson method combined with constant arc length method is used for the solution of nonlinear equilibrium equations. The zero value of the tangent stiffness matrix determinant of the structure is used as the criterion of the buckling state. A parabolic interpolation method [11] of the arc length is used to determine the buckling load. In order to initiate the secondary path, a perturbation displacement proportional to the first buckling mode is added at the bifurcation point.

Numerical examples are studied to investigate the effect of bimoment and the prebuckling twist deformations on the critical state, the failure mode of the global buckling and postbuckling behavior of axially loaded point symmetric thin-walled open section beams with different section geometries, lengths and boundary conditions.

2. Finite element formulation

In this paper, only the point-symmetric open section is considered. Thus, the centroid and the shear center of the cross section are coincident. The beam element developed here has two nodes with seven degrees of freedom per node. The element nodes are chosen to be located at the centroids of the end cross sections of the beam element, and the centroid axis is chosen to be the reference axis of the beam element. The CR formulation proposed in [10] and the kinematics of the beam element proposed in [11] are adapted and used in this paper. For completeness, clarity, and ease of reference, the derivation of the internal nodal force vector and the tangent stiffness matrix given in [10] are briefly repeated with a small modification.

2.1. Basic assumptions

The following assumptions are made in the derivation of behavior of the thin-walled beam element with point-symmetric open section.

(1) The beam is straight, prismatic and slender, and the Euler-Bernoulli hypothesis is valid, when the out-of-plane warping of the cross section is excluded.
(2) The unit extension of the centroid axis of the beam element is uniform.
(3) The cross section of the beam element does not deform in its own plane and strains within this cross section can be neglected.
(4) The out-of-plane warping of the cross section is the product of the twist rate of the beam element and the Saint Venant warping function for a prismatic thin walled beam of the same cross section.
(5) The deformation displacements and rotations of the beam element are small.
(6) The material is homogeneous, isotropic and linear elastic.

In conjunction with the CR formulation, the fifth assumption can always be satisfied if the element size is properly chosen. The assumption of small strains may not be required in this study. However, the material is assumed to be linear elastic. The yield strains for most engineering materials are small. Thus, only small strains are considered here. In this study, Prandtl's membrane analogy and the Saint Venant torsion theory [13] are used to obtain an approximate Saint Venant warping function for a prismatic thin walled beam.

2.2. Coordinate systems

In order to describe the system, we define three sets of right handed rectangular Cartesian coordinate systems:

(1) A fixed global set of coordinate system, $\mathbf{X}^G = \{X_1^G \quad X_2^G \quad X_3^G\}$ (see Fig. 1); the nodal coordinates, nodal displacements and rotations, the equilibrium equations, and the stiffness matrix of the system are defined in this coordinate system.
(2) Element cross section coordinate system, $\mathbf{x}^S = \{x_1^S \quad x_2^S \quad x_3^S\}$ (see Fig. 1); a set of element cross section coordinates is associated with each cross section of the beam element. The origin of this coordinate system is rigidly attached to the centroid of the cross section. The x_1^S axis is chosen to coincide with the normal of the

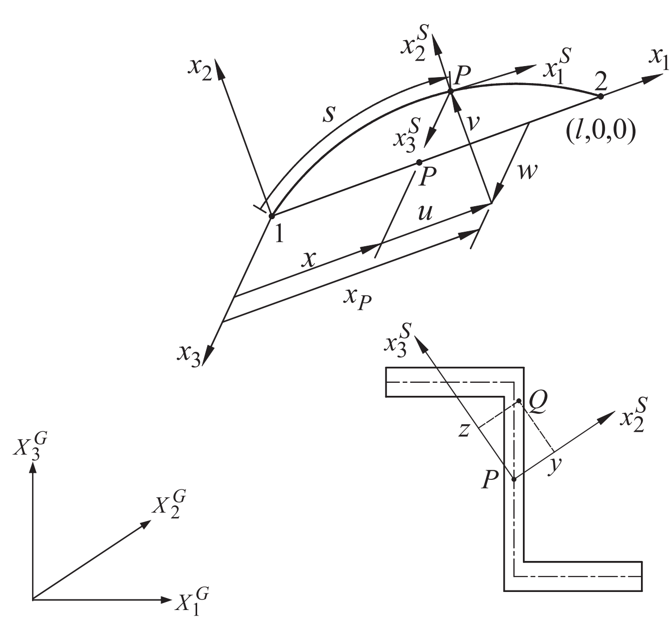

Fig. 1. Coordinate systems.