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Nonlinear behaviour of short elastic cylindrical shells under global bending

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ABSTRACT

A recent computational study identified four distinct domains of stability behaviour at different lengths in thin elastic cylindrical shells under global bending. Cylinders of sufficient length suffer from fully-developed cross-sectional ovalisation and fail by local buckling at a moment very close to the Brazier prediction. Progressively shorter cylinders experience less ovalisation owing to the increasingly strong restraint provided by the boundary at the edges. Very short thin cylinders, however, restrain the formation of even a local buckle and fail through a limit point instability at moments and curvatures significantly in excess of the classical elastic prediction. This limit point behaviour is not caused by ovalisation but by the growth of a destabilising fold on the compressed meridian.

The nonlinear behaviour of very short cylinders under global bending is investigated in detail herein, covering a wide range of lengths, radius to thickness ratios and boundary conditions with both restrained and unrestrained meridional rotations corresponding to 'clamped' and 'simply-supported' conditions respectively. Two types of imperfections are investigated, the critical buckling eigenmode and a realistic manufacturing-related 'weld depression'. A complex insensitivity to these imperfections is revealed owing to a pre-buckling stress state dominated by local compatibility bending, and the cylinder length is confirmed as playing a crucial role in governing this behaviour. The study contributes to the characterisation of multi-segment shells with very short individual cylindrical segments, often found in the aerospace and marine industries as well as in specialised civil engineering applications such as LIPP® silos.

1. Introduction

The nature of the elastic stability of a cylindrical shell under the fundamental loads of uniform axial compression, external pressure and torsion is known to be strongly controlled by its length. During design, cylinders are therefore first categorised into a length domain and dimensioned according to procedures that capture the buckling behaviour governing that particular domain. Each one of the above classical load cases qualitatively exhibits three length domains, namely 'short', 'medium' and 'long' [1–6], though the length boundaries defining each domain occur at different numerical values for each load case.

For 'short' perfect cylindrical shells under uniform axial compression (Fig. 1), the end boundary condition induces extensive compatibility bending deformations and completely restrains the formation of any local buckling mode. Failure occurs by global limit point buckling at a considerably higher buckling stress than that predicted by the classical elastic critical buckling stress σ_{cl} (Eq. (1)), derived assuming a pure pre-buckling membrane stress state achievable only in a theoretical cylinder with special boundary conditions:

$$\sigma_{cl} = \frac{E}{\sqrt{3(1-\nu^2)}}\cdot\frac{t}{r} \approx 0.605E\frac{t}{r} \text{ for } \nu = 0.3. \tag{1}$$

Here E is the Young's modulus and ν is the Poisson's ratio. In the 'medium' length domain, by contrast, the effect of the boundary restraint is localised near the ends and no longer sufficient to restrain the formation of a local short-wave buckling mode. Since the cylinder is now largely under membrane action, buckling occurs at a stress only slightly below σ_{cl}. The approximately 15% reduction is caused by geometric nonlinearity due to the pre-buckling amplification of compatibility bending rotations near the end boundaries [2,7]. 'Long' cylinders fail in a different manner usually by Euler column buckling at a stress significantly below σ_{cl}.

These three domains are illustrated in Fig. 1 [1] where the dimensionless length is expressed in terms of the Batdorf [8] parameter Z, where:

$$Z = \frac{L^2}{rt}\sqrt{1-\nu^2} = \omega^2\sqrt{1-\nu^2}. \tag{2}$$

This choice of normalisation has since been superseded by a related but simpler parameter ω [2], where:

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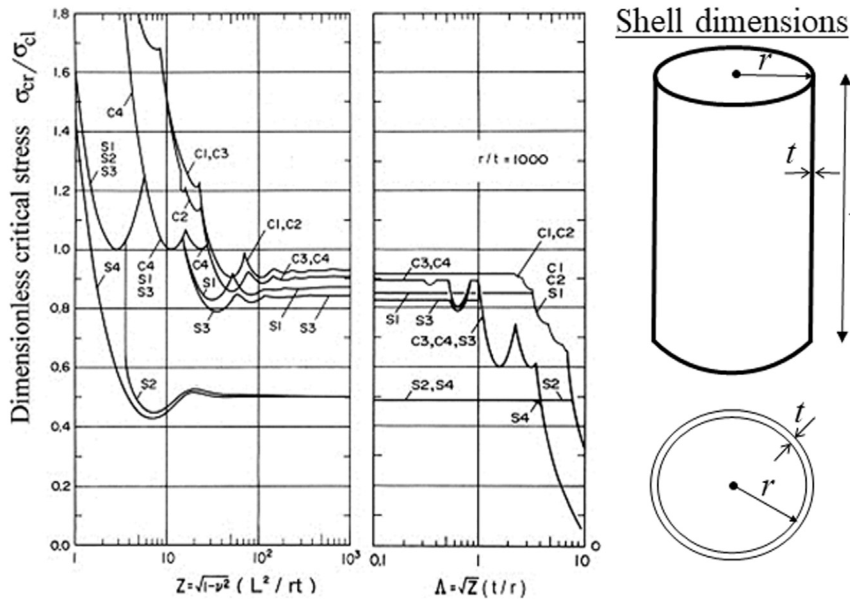

Fig. 1. Effect of length and end boundary conditions on the buckling strength of a perfect cylindrical shell with $r/t = 1000$ under uniform axial compression, where r is the radius to middle surface, t is the thickness of the wall and L is the length of the shell (from [1]).

$$\omega = \frac{L}{\sqrt{rt}} = \frac{\sqrt{Z}}{\sqrt[4]{1-\nu^2}}, \tag{3}$$

which is directly related to the linear bending half-wavelength λ, defined as:

$$\lambda = \frac{\pi\sqrt{rt}}{[3(1-\nu^2)]^{0.25}} \approx 2.444\sqrt{rt} \;\text{ for }\; \nu = 0.3, \tag{4}$$

and the classical axisymmetric buckle half-wavelength λ_{cl}, given by:

$$\lambda_{cl} = \frac{\pi\sqrt{rt}}{[12(1-\nu^2)]^{0.25}} \approx 1.728\sqrt{rt} \;\text{ for }\; \nu = 0.3. \tag{5}$$

The parameter ω is linear in the cylinder length L and permits the influence of the shell slenderness, associated with the r/t ratio, to be examined independently of the specific cylinder length within each length domain. Using this notation, the European Standard for metal shells EN-1993-1-6 [5] defines the length ranges for cylindrical shells under uniform axial compression in a remarkably compact manner as simply $\omega \le 1.7$ for 'short' cylinders, $1.7 < \omega \le 0.5(r/t)$ for 'medium' and $\omega > 0.5(r/t)$ for 'long' cylinders. External pressure and torsion are characterised similarly. The value of $\omega = 1.7$ for the 'short-medium' boundary corresponds to $Z = 2.85$ in Fig. 1 and is a conservative choice to cover all possible edge boundary conditions. In reality, however, it may be seen that under clamped conditions (with restrained meridional edge rotations, denoted in Fig. 1 by 'C') the 'short' domain persists over a wider range of lengths than under simply-supported conditions (with unrestrained edge rotations, denoted by 'S').

2. Buckling at different lengths under global bending

Building on this theory, a recent computational study of perfect clamped cylindrical shells under uniform bending by Rotter et al. [9] characterised the effect of length on the nonlinear elastic stability across a wide range of r/t ratios. The study established that the behaviour under global bending displays length domains that are qualitatively similar to those of the three fundamental load cases, except for the presence of an additional 'transitional' region between the 'medium' and 'long' domains (Fig. 2). This new domain is a direct consequence of the Brazier [10] cross-section ovalisation phenomenon, not present in the other load cases, that detrimentally affects the pre-buckling deformations of 'long' cylinders under bending. Specifically, the 'transitional' domain identifies the region of lengths below which ovalisation is completely restrained by the end boundaries, and above which ovalisation is fully developed and cannot become more detrimental. The Brazier moment M_{Braz}, defined by:

$$M_{Braz} = \frac{2\sqrt{2}}{9} \cdot \frac{E\pi r t^2}{\sqrt{1-\nu^2}} \approx 1.035Ert^2 \approx 0.54M_{cl} \;\text{ for }\; \nu = 0.3, \tag{6}$$

describes limit point cross-sectional failure mode of long cylinders through nonlinear ovalisation and is only approximately half the moment M_{cl}, where:

$$M_{cl} = \pi r^2 t \sigma_{cl}(r) \approx 1.901Ert^2 \approx 1.84M_{Braz} \;\text{ for }\; \nu = 0.3, \tag{7}$$

that describes local buckling of shorter cylinders, a significant penalty.

The interested reader is invited to consult any of [11–13] for an in-depth discussion of the nonlinear mechanics of ovalisation.

As far as the authors are aware, the study by Rotter et al. [9] also appears to have been the first to confirm formally and briefly explore the existence of a 'short' domain for cylinders under bending. For clamped end conditions, the domain boundary between 'short' and 'medium' cylinders was deemed to occur at $\omega = 4.8$, somewhat longer than the value of $\omega = 1.7$ for the closely related load case of uniform axial compression. Fajuyitan et al. [14] later extended this work to simply-supported end conditions and found that this boundary then fell at a shorter value of $\omega = 3.2$ due to the weaker rotational restraint, consistent with the effect seen for uniform axial compression in Fig. 1. In both cases, cylinders in the 'short' domain were found to exhibit a limit point instability at moments and mean curvatures significantly in excess of the classical elastic critical buckling values M_{cl} and φ_{cl} (Eqs. (7) and (8)) derived on the basis of the simple 'local buckling hypothesis' [11,15–17]. This hypothesis assumes that a local short-wave buckle develops as soon as the most compressed fibre in the cylinder reaches the classical elastic critical stress for uniform axial compression σ_{cl}, presented in Eq. (1). This concept is clearly no longer valid for the 'short' domain where the boundary condition restraint entirely eliminates this local buckling mode. Further, as is explored in this study, the limit point instability in this length domain is not attributable to Brazier ovalisation but rather to the progressive growth of a destabilising meridional fold [18] on the compressed side of the cylinder (Fig. 3),

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