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Shape and size optimization of functionally graded sandwich plates using isogeometric analysis and adaptive hybrid evolutionary firefly algorithm

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ABSTRACT

The paper presents an effective methodology for modeling and simultaneously optimizing the layer thicknesses (shape) and the ceramic volume fraction distribution (size) of functionally graded (FG) sandwich plates under free vibration in the framework of isogeometric analysis (IGA). The multi-patch B-spline basis functions separately defined in each of the layer thicknesses are used to represent the ceramic volume fraction distribution. Accordingly, the C^0– continuity at layer interfaces can be naturally satisfied without any additional conditions. Furthermore, this multi-patch B-spline representation still ensures the continuously and smoothly varying material properties across each layer thickness. The effective material properties are then estimated by either the rule of mixture or the Mori-Tanaka scheme. A non-uniform rational B-splines (NURBS)-based isogeometric finite element model associated with the third-order shear deformation theory (TSDT) is utilized for the plate free vibration analysis. A recently developed adaptive hybrid evolutionary firefly algorithm (AHEFA) with the improvement on the convergence speed and the solution accuracy is employed as an optimizer. Design variables are the layer thicknesses and the ceramic volume fractions at control points located in the thickness direction. Several numerical examples of two types of optimization problems of the FG sandwich plates, including (i) the first natural frequency maximization with volume constraints, and (ii) mass minimization with frequency constraints, are presented to illustrate the effectiveness and reliability of the proposed method.

1. Introduction

Known as one of the advanced composite materials was first delivered by a group of Japanese material scientists in 1984 [1], functionally graded materials (FGMs) are a mixture of at least two distinct constituents engineered to possess the continuously and smoothly altering macroscopic material properties. As seen, the ceramic-metal composite is one of the most commonly studied FGMs owing to superior mechanical merits yielded from their synergy. Indeed, the ceramic phase works very well in fairly high temperature environments, but it is of a low tensile strength, whereas the metallic one has prominent fracture toughness. Additionally, the gradual spatial variation of material properties effectively prevents the sudden transitions of stress fields that may cause delamination and lead to the reduced overall stiffness, even undesired failures of the whole structure. Thanks to the prominent mechanical advantages mentioned above, this material has received remarkable attention of many researchers and been extensively developed to various practical engineering fields. Another salient extension of the above material is the ceramic-metal FG sandwich one which has been very potential to structural applications such as beam, plate and shell, etc. Two most popularly referred types of FG sandwich models often found in such structures are: (i) the first model with a FGM core layer, a homogeneous metallic bottom layer and a homogeneous ceramic top layer, and (ii) the second model with a homogeneous ceramic core layer and two FGM face layers. Due to their wide applications, a large number of studies have been performed to investigate its behavior for a more profound knowledge, including: beams [2–7], plates [8–14], shells [15–20], and so forth. A comprehensive overview of research on FG plates and shells was reported in Refs. [21,22].

As an alternative to the standard FEM in finding advanced numerical methods, isogeometric analysis (IGA) developed towards the integration of Computer Aided Design (CAD) and finite element analysis (FEA) into a single model was first introduced by Hughes et al. [23]. Interested readers are encouraged to refer to the excellent book [24] for a more detailed discussion. The core idea of this approach is to use the same NURBS basis functions for describing geometry as well as approximating unknown fields in FEA. In comparison with the standard FEM, the IGA has the prominent advantages as follows: (i) describe complex geometry exactly; (ii) fulfill high-order derivatives and

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continuity of the NUBRS basis functions easily; and (iii) decrease the number of degree of freedoms (DOFs) for high-order elements significantly. In addition, the overall cost CAD-FEM conversion operations can be considerably saved, especially for problems with complex geometry. Therefore, the IGA has commonly become and widely accepted in the analysis community with a large number of outstanding papers published during the past years, especially for concerns relating to behavior analyses of FG plates [25–27], and various fields [28–35], etc.

In all the foregoing work, the ceramic volume fraction distributions are often given by a predefined mathematical function with gradient indexes such as the power and exponential laws [36]. The desired effective material properties can be readily achieved by adjusting those indexes. This aims to reduce the possible complexities in modeling the material distributions as well as to facilitate the manufacturing process. Nevertheless, such functions are fairly subjectively given by design engineers without taking account of individual properties of every practical problem. Therefore, the volume fraction distribution for tailoring a FGM structure should be optimized to meet the desired implementation under certain constraints. Accordingly, the selection of an appropriate optimization algorithm to solve such problems is really essential, especially for ones that the natural frequencies and constraints are highly nonlinear, non-convex and greatly sensitive to the switch between mode shapes during the optimization process. Although gradient-based algorithms quickly converge to the optimal solution and can significantly save the computational cost. However, these methods always demand sensitivity analyses of the objective function and constraints whose performances are relatively expensive and complex, even impossible in many cases. Furthermore, optimized solutions may be gotten stuck in local regions since these algorithms only exploit a given set of initial values in a specified search space based on the derivative information obtained from sensitivity analyses. To tackle these issues, non-gradient-based algorithms, known as metaheuristic approaches, have been quickly developed and extensively applied to various engineering problems as an effective alternative. Because these methods utilize stochastic searching techniques to randomly select potential solutions within a predefined search space, they extremely eliminate sensitivity analyses as well as considerably reduce the mathematical complexities. Accordingly, a global optimal solution can be achieved, but the time-consuming process increases since none of oriented information is provided to the search process over the given whole feasible region. Among them, differential evolution (DE) algorithm first developed by Storn and Price [37] has shown its effectiveness in numerous engineering applications [38]. Another optimization approach as a generalization to the particle swarm optimization (PSO) [39], simulated annealing (SA) [40], and DE algorithms proposed by Yang [41–43] is the so-called firefly algorithm (FA). This algorithm inherits the advantages of all the above three ones and can effectively perform in numerous cases [44]. However, the convergence rate and the solution accuracy of both approaches still need to be further enhanced. Therefore, a large number of improved algorithms and their variants have been released, interested readers could consult Refs. [38,44] for a more detailed discussion. Recently, a novel metaheuristic algorithm as a hybridization of the DE method and the FA which is called the adaptive hybrid evolutionary firefly algorithm (AHEFA) was developed by Lieu et al. [45]. This method has proven its effectiveness and robustness in enhancing the convergence speed and the accuracy of obtained optimal solution compared with many other approaches to shape and size optimization problems of truss structures with multiple frequency constraints. Its application to material distribution optimization of FG plates under thermo-mechanical loads has been subsequently studied by Lieu and Lee [46] as well.

As seen, a large number of papers concerning the material distribution optimization of FG structures have published during past decades. For example, Tanaka et al. [47–49] presented a methodology for designing thermoelastic material of FG hollow cylinders using the direct sensitivity analysis and optimization techniques. Turteltaub [50,51] used the line-search step of a conjugate gradient method to determine an optimal material layout of bi-directional FG heat conduction problems. Cho and Ha [52,53] used piecewise bilinear Lagrangian shape functions to represent the volume fraction distribution via a finite number of nodes whose coordinates are predefined in a 2D domain. This description is always C^0- continuous across element boundaries due to the inherent attributes of the interpolation functions. An interior penalty-function method and a sensitivity analysis based on the finite difference scheme were utilized to solve constrained optimization problems. Chen and Tong [54,55] presented the thermo-mechanically coupled sensitivity analysis and design optimization of bi-directional FG elasticity structures. In which, volume fractions at nodes discretized in a computational domain are approximated as those done by Cho and Ha [52,53]. Problem formulations were then handled by the standard FEM and the sequential linear programming (SLP). Goupee and Vel [56–58] employed the piecewise Hermite polynomial functions for the volume fraction representation of bi-directional FG elasticity structures. Accordingly, the material distribution is always C^1- continuous over the whole design domain. However, these cubic splines are pointwise negative over some parts of a given interval, the physical boundary constraint of design variables within [0,1] may therefore be violated if a necessary and sufficient condition on the positivity of these functions is not imposed closely as proved in Refs. [59,60]. Another limitation in fulfilling desired orders of continuity for the material distribution is observed as well. The genetic algorithm (GA) and element-free Galerkin were used in their numerical experiments. Ashjari and Khoshravan [61] used the piecewise Hermite interpolation polynomials to describe the ceramic volume fraction distribution via a set of equally given points in the thickness direction of the FG plate such that its mass is minimized under deflection and stress constraints for static loads. The authors utilized the Navier approach for the analysis, and the real-coded GA and PSO algorithm to solve optimization problems. Roque and Martins [62] maximized the first frequency of the FG beams by optimizing the ceramic volume fraction distribution along its height based on a three-parameter power law. Although the number of design variables given by this law decreases, a generalization of applying to many practical engineering problems appears to be restricted because its material distribution is idealized by a predefined mathematical function. In that work, the authors used the meshless numerical method for the free vibration analysis, and the DE algorithm as an optimization tool. Kamarian et al. [63] used the FA and Adaptive Neuro-Fuzzy Inference System (ANFIS) to optimize the volume fraction distribution of three-parameter FG beams resting on elastic foundations. Yas et al. [64] applied an imperialist competitive algorithm and neural networks to the same above problem in their experiments. More recently, Tsiatas and Charalampakis [65] utilized the DE algorithm for optimizing the material distribution of axially FG beams and arches based on a four-parameter power law and five-parameter trigonometric functions, while the analog equation method (AEM) was employed for the free vibration analysis. Several optimization problems regarding the frequency maximization and mass minimization under frequency constraints were examined. Numerous other considerable studies on this topic could be found in Refs. [66–75], etc. As can be seen that IGA applications to the material distribution optimization of FGMs appear to be limited aside from a few recently published papers. In particular, Taheri and Hassani [76] employed the IGA and the sequential quadratic programming (SQP) for the thermo-elastic material distribution optimization of 2D elasticity problems. Taheri and Hassani [77] also utilized the same above methodology to maximize the frequencies of 2D elasticity structures by simultaneously designing their shape and material.

To the best knowledge of the authors, no paper concerning modeling and simultaneously optimizing the layer thicknesses (shape) and the ceramic volume fraction distribution (size) of the FG sandwich plates under free vibration within the framework of the IGA has been published so far, especially for the volume fraction representation based