Thin-Walled Structures 123 (2018) 162–184

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

On the mechanics of distortional-global interaction in fixed-ended columns

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ARTICLE INFO

Keywords:
Cold-formed steel fixed-ended columns
Lipped channels
Zed-sections
Distortional-global (D-G) interaction
Generalised Beam Theory (GBT)
Geometrically non-linear imperfect analysis (GNIA)
D-flexural/torsional/distortional coupling
D-flexural coupling
D-torsional coupling

ABSTRACT

This work reports the results of a numerical investigation concerning the elastic post-buckling behaviour of fixed-ended thin-walled lipped channel and zed-section columns affected by distortional-global (D-G) interaction – "unexpected" features associated with the global buckling nature are unveiled in both cases. The columns analysed (i) exhibit cross-section dimensions and lengths ensuring the simultaneous occurrence of distortional and global critical buckling, and (ii) contain critical-mode initial geometrical imperfections (linear combinations of the critical distortional and global buckling mode shapes). For comparison and clarification purposes, the post-buckling behaviour of lipped channel columns buckling in an "isolated" distortional mode and containing "pure" distortional and global (flexural-torsional) initial geometrical imperfections are also analysed. The results presented and discussed are obtained through geometrically non-linear Generalised Beam Theory (GBT) analyses and provide the evolution, along given equilibrium paths, of the column deformed configuration (expressed in GBT modal form), relevant displacement profiles and modal participation diagrams, making it possible to acquire in-depth insight on the mechanics underlying D-G interaction in columns. Finally, particular attention is paid to interpreting the differences exhibited by the various column post-buckling behaviours investigated.

1. Introduction

Cold-formed steel (CFS) open-section thin-walled members, namely columns, exhibit geometries (cross-section dimensions and lengths) that often make them highly susceptible to several instabilities involving individual (local, distortional, global – L, D, G) and/or coupled (L-D, L-G, D-G or L-D-G) deformation patterns. Their efficient design constitutes a very complex task (when compared with the hot-rolled steel members), since mode interaction phenomena may occur even when the associated critical buckling loads are significantly apart – contrarily to the general belief that these coupling phenomena only develop when the competing buckling loads are nearly coincident (or very close). Therefore, in order to assess the structural response of such members it does not suffice to acquire in-depth knowledge about their "pure" (individual) buckling and post-buckling behaviours. Indeed, it is indispensable to account for the possible occurrence of several mode coupling effects, which may erode, to a smaller or larger extent (depending on the slenderness), the member ultimate strength – failing to do it may lead to a high likelihood of reaching unsafe designs.

The existing studies on interaction phenomena involving distortional buckling in CFS columns (e.g., Camotim et al. [1]), which comprise experimental investigations, numerical simulations and/or design proposals, are clearly "unbalanced" since most of them concern L-D interaction – in this regard, it is worth mentioning the works of Kwon and Hancock [2], Young et al. [3] and Martins et al. [4–6]. To the authors' best knowledge, the investigations addressing the influence of D-G interaction on the post-buckling behaviour and ultimate strength of CFS columns consist of (i) experimental investigations on rack-section uprights with and without holes [7–9], lipped channel columns [10] and web-stiffened lipped channel columns [11] (it is still worth noting the tests on cold-formed stainless steel lipped channel columns reported by Rossi et al. [12,13]), and (ii) the numerical (shell finite element) investigations on simply supported (locally/globally pinned end cross-sections with free or prevented warping) and fixed-ended lipped channel columns [14,15]), and simply supported and warping-prevented rack-section uprights with or without holes [16]. Recently, the authors carried out an investigation on the relevance and Direct Strength Method (DSM) design of lipped channel, web-stiffened lipped channel and zed-section columns undergoing D-G interaction [17]. However, an investigation focusing on the mechanics underlying this coupling phenomenon is still lacking. This work aims at contributing towards filling the above gap, by using a Generalised Beam Theory (GBT) approach to assess the mechanical features associated with the post-buckling behaviour of columns affected by D-G interaction, thus extending available results dealing with such columns [18–20]. Therefore, the results presented and discussed in this work are obtained

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https://doi.org/10.1016/j.tws.2017.11.001
Received 17 August 2017; Received in revised form 25 October 2017; Accepted 2 November 2017
Available online 22 November 2017
0263-8231/

by means of a geometrically non-linear imperfect analyses (GNIA) performed with GBT, a one-dimensional bar theory for prismatic thin-walled members that combines the accuracy of shell finite element models with clarifying modal solutions providing in-depth insight on the mechanics of the problem under consideration (e.g., [21]). This feature makes GBT-based GNIA ideally suited to investigate mode interaction problems, which involve deformation patterns of various natures.

This work addresses the elastic post-buckling behaviour of fixed-ended lipped channel and zed-section columns undergoing D-G interaction. The columns analysed (i) exhibit cross-section dimensions and lengths ensuring practically coincident distortional and global critical buckling loads (the so-called "true D-G interaction" [17]), and (ii) contain critical-mode initial geometrical imperfections obtained as linear combinations of the distortional and global buckling modes, provided by preliminary GBT buckling analyses. For comparison and clarification purposes, the post-buckling behaviours of lipped channel columns buckling in "isolated" distortional modes and containing "pure" distortional or global (flexural-torsional) initial geometrical imperfections are also analysed. The GBT-based GNIA results, validated through the comparison with values yielded by ABAQUS [22] shell finite element analyses (SFEA), provide the evolution, along given equilibrium paths, of the column deformed configuration (expressed in GBT modal form), relevant displacement profiles and stress distributions, making it possible to acquire in-depth insight on the column D-G interaction mechanics. Finally, particular attention is devoted to interpreting the differences exhibited by the several aforementioned column post-buckling behaviours.

2. Review of the GBT geometrically non-linear formulation

A typical GBT-based analysis involves two (independent) main tasks: (i) a *cross-section analysis*, leading to the determination of the deformation modes and evaluation of the corresponding modal mechanical properties, and (ii) a *member analysis* (elastic buckling and post-buckling analyses, in this work) – the main steps and hypotheses of the geometrically non-linear GBT formulation developed by the authors [23] is briefly reviewed next. Then, a description of the deformations modes most relevant to the analysis of the lipped channel and zed-section columns undergoing D-G interaction is provided.

2.1. Formulation, finite element approximation and solution procedure

The member mid-surface displacement field is expressed as products of two one-dimensional functions, namely (i) the deformation mode shapes ($u_k(s)$, $v_k(s)$ and/or $w_k(s)$ – obtained from the cross-section analysis, briefly discussed in Section 2.2) and (ii) the modal amplitude functions $\phi_k(x)$, which constitute the member analysis unknowns. One then has

$$u(x,s)=u_k(s)\phi_{k,x}(x) \tag{1.1}$$

$$v(x,s)=v_k(s)\phi_k(x) \tag{1.2}$$

$$w(x,s)=w_k(s)\phi_k(x) \tag{1.3}$$

where (i) $(.)_{,x}\equiv d(.)/dx$ and (ii) Einstein's summation convention applies to subscript k – the initial geometrical imperfections incorporated in the analyses (discussed in Section 3) follow the same trends. The kinematic (strain-displacement) relationships adopted are similar to those considered in [24,25] (GBT and finite strip non-linear analyses, respectively) and read

$$\varepsilon_{xx}=u_{,x}-zw_{,xx}+\frac{1}{2}(u_{,x}^2+v_{,x}^2+w_{,x}^2)-\bar{u}_{,x}+z\bar{w}_{,xx}-\frac{1}{2}(\bar{u}_{,x}^2+\bar{v}_{,x}^2+\bar{w}_{,x}^2) \tag{2.1}$$

$$\varepsilon_{ss}=v_{,s}-zw_{,ss}+\frac{1}{2}(u_{,s}^2+v_{,s}^2+w_{,s}^2)-\bar{v}_{,s}+z\bar{w}_{,ss}-\frac{1}{2}(\bar{u}_{,s}^2+\bar{v}_{,s}^2+\bar{w}_{,s}^2) \tag{2.2}$$

$$\gamma_{xs}=u_{,s}+v_{,x}-2zw_{,xs}+u_{,x}u_{,s}+v_{,x}v_{,s}+w_{,x}w_{,s}-\bar{u}_{,s}-\bar{v}_{,x}+2z\bar{w}_{,xs}-\bar{u}_{,x}\bar{u}_{,s}-\bar{v}_{,x}\bar{v}_{,s}-\bar{w}_{,x}\bar{w}_{,s} \tag{2.3}$$

The member strain energy, expressed in terms of the strain and stress components, is given by

$$U=\frac{1}{2}\int_V\sigma_{ij}\varepsilon_{ij}dV=U_1+U_2+U_3-(\bar{U}_1+\bar{U}_2+\bar{U}_3) \tag{3}$$

where U_i and $\bar{U}_i$ (i = 1, 2, 3) are the strain energy terms associated with the total deformation and initial geometric imperfections, respectively – see the details in [23].

The modal amplitude functions appearing in (3) are approximated through 4-node beam finite elements based on approximation functions that are linear combinations of:

(i) Lagrange cubic polynomial primitives $\psi_i^L(\xi)$, $i=1,\ldots,4$, for the (warping only) axial extension and shear modes, which read ($d_{k1}=\phi_{k,x}(0)$, $d_{k2}=\phi_{k,x}(L_e/3)$, $d_{k3}=\phi_{k,x}(2L_e/3)$, $d_{k4}=\phi_{k,x}(L_e)$)

$$\phi_{j,x}(x)=\psi_{1,x}^L d_{j1}+\psi_{2,x}^L d_{j2}+\psi_{3,x}^L d_{j3}+\psi_{4,x}^L d_{j4} \tag{4.1}$$

(ii) Hermite cubic polynomials $\psi_i^H(\xi)$, $i=1,\ldots,4$, for the remaining (conventional and transverse extension) deformation modes,[1] given by ($d_{k1}=\phi_{k,x}(0)$, $d_{k2}=\phi_k(0)$, $d_{k3}=\phi_{k,x}(L_e)$, $d_{k4}=\phi_k(L_e)$)

$$\phi_k(x)=\psi_1^H d_{k1}+\psi_2^H d_{k2}+\psi_3^H d_{k3}+\psi_4^H d_{k4} \tag{4.2}$$

The finite element internal force vector components concerning deformation mode h and d.o.f. α ($f_{h\alpha}^e$) is obtained by incorporating (4.1)–(4.2) into the strain energy (3) and differentiating w.r.t. $d_{h\alpha}$ – it reads

$$f_{h\alpha}^e=\frac{\partial U}{\partial d_{h\alpha}}=f_{h\alpha}^1+f_{h\alpha}^2+f_{h\alpha}^3-(\bar{f}_{h\alpha}^1+\bar{f}_{h\alpha}^2+\bar{f}_{h\alpha}^3) \tag{5.1}$$

where, after some manipulation,

$$f_{h\alpha}^1=T_{hi\alpha\beta}^1 d_{i\beta} \tag{5.2}$$

$$f_{h\alpha}^2=\frac{1}{2}T_{hi\alpha\beta}^2 d_{i\beta} \tag{5.3}$$

$$f_{h\alpha}^3=\frac{1}{3}T_{hi\alpha\beta}^3 d_{i\beta} \tag{5.4}$$

$$\bar{f}_{h\alpha}^1=T_{hi\alpha\beta}^1\bar{d}_{i\beta} \tag{5.5}$$

$$\begin{aligned}\bar{f}_{h\alpha}^2 = {} & \bar{T}_{hi\alpha\beta}^2 d_{i\beta}+\Big(\frac{1}{2}C_{hij}^I k_{\alpha\beta\eta}^{211}+\frac{1}{2}B_{hij}^I k_{\alpha\beta\eta}^{000}+D_{hij}^I k_{\alpha\beta\eta}^{110}\\ &+\frac{1}{2}E_{hij}^I k_{\alpha\beta\eta}^{200}+\frac{1}{2}(E_{hij}^{II}+B_{hij}^{II})k_{\alpha\beta\eta}^{011}+\frac{1}{2}C_{hij}^{II}k_{\alpha\beta\eta}^{222}\\ &+D_{hij}^{II}k_{\alpha\beta\eta}^{121}+\frac{1}{2}E_{hij}^{III}k_{\alpha\beta\eta}^{022}+\frac{1}{2}E_{ijh}^{IV}k_{\beta\eta\alpha}^{112}\Big)\bar{d}_{i\beta}\bar{d}_{j\eta}\end{aligned} \tag{5.6}$$

$$\bar{f}_{h\alpha}^3=\bar{T}_{hi\alpha\beta}^3 d_{i\beta} \tag{5.7}$$

and

(i) $T_{hi\alpha\beta}^1$, $T_{hi\alpha\beta}^2$, $T_{hi\alpha\beta}^3$, $\bar{T}_{hi\alpha\beta}^2$, $\bar{T}_{hi\alpha\beta}^3$ are the components of the tangent stiffness matrix (details in [23]) – the elementary tangent stiffness matrix components are given by ($\bar{T}_{hi\alpha\beta}^1=0$)

$$T_{hi\alpha\beta}^e=T_{hi\alpha\beta}^1+T_{hi\alpha\beta}^2+T_{hi\alpha\beta}^3-(\bar{T}_{hi\alpha\beta}^2+\bar{T}_{hi\alpha\beta}^3) \tag{6}$$

[1] The various deformation mode families are addressed in Section 2.2.