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Vibrations of compound shells of revolution with elliptical toroidal members

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A B S T R A C T

Vibrations of thin-walled systems compound of coaxial conjugate shells of revolution of different shapes with torus-elliptical members are analyzed. The shells can be composed of one or several layers, of isotropic and orthotropic materials with variable geometric and stiffness characteristics along a generatrix-meridian. Small undamped vibrations of such systems are studied using the classical Kirchhoff-Love theory. To solve the appropriate eigen-value two-dimensional problems, the numerical-analytical technique, which includes the Fourier variable-separation method, incremenal search method ($\Delta(\lambda)$ -method), and the orthogonal sweep method with solving Cauchy's problems by the fifth-order Runge-Kutta scheme, is developed. It is shown by a number of examples that vibrations of the shell system as a single whole have qualitative features in comparison with vibrations of its separate members.

1. Introduction

Many structures of modern engineering, such as bodies of rockets and submarines, oil and gas reservoirs, protective shelters for nuclear reactors, gatherings (adapters), are modeled by thin-walled systems of conjugated coaxial shells of revolution with various geometrical forms. To track resonance service regimes of such systems under the action of real loads and prevent arising of emergencies, it is necessary to know their dynamic characteristics.

The literature devoted to the analysis of natural frequency vibrations of compound shells and to development of methods for solving associated (respective) eigenvalue problems is quite voluminous [1–15,17,21–24].

Such vibrations are the most completely studied as applied to the systems composed of cylindrical and conical shells with zero Gaussian curvature as well as of circular plates.

So, in [1], the resonance frequencies of a cylinder - truncated cone system of isotropic material are studied experimentally and analytically depending on the taper angle of the conical member (element, part). The same isotropic shell system under various boundary conditions was considered in [2] within the framework of the Donnell-Mushtary-Flugge equations using the series method to approximate the displacement function of the conical part.

In [3], the dynamic characteristics of a laminated structure composed of a cylinder and two cones modeling the fragment of a spacecraft are analyzed. A thin-walled cone-cylinder shell system was modeled in [4] within the framework of the transfer matrix approach by implementing the Flugge equations of motion. More recently, in [5], the problem was solved using shell finite element, for which the variational quantities were calculated using the Mindlin theory. In [6], the results obtained making use finite element method (FEM) for a thin-walled cone-cylinder shell system were verified by comparing with experimental ones. In [7], free vibration characteristics for a cone-cylinder shell system of variable thickness were determined by the Ritz method.

Free vibrations of a structure consisting of a finite circular cylindrical shell closed at one end by a circular plate were analyzed in [8] using the variational method. In [9], the natural frequencies and mode shapes for a conical shell with ends closed by annular or round plates are investigated in detail by combining the vibration theory with the transfer matrix method.

The paper [10] presents a modified variational method for dynamic analysis of circumferentially ribbed cone-cylinder shell systems under various boundary conditions. Double mixed series, i.e. the Fourier series and Chebyshev orthogonal polynomials, are adopted as admissible displacement functions for each shell segment.

The geometric complication of compound shell systems is attributed to including spherical members with nonzero but constant Gaussian curvature. This makes it possible, unlike the systems with zero Gaussian curvature, to essentially extend the class of objects being considered including high pressure balloons, reservoirs of various designations, protective shelters.

The paper [11] presents the analytical solution of the problem on vibrations of a sphere-cylinder-sphere system that models dynamics of a

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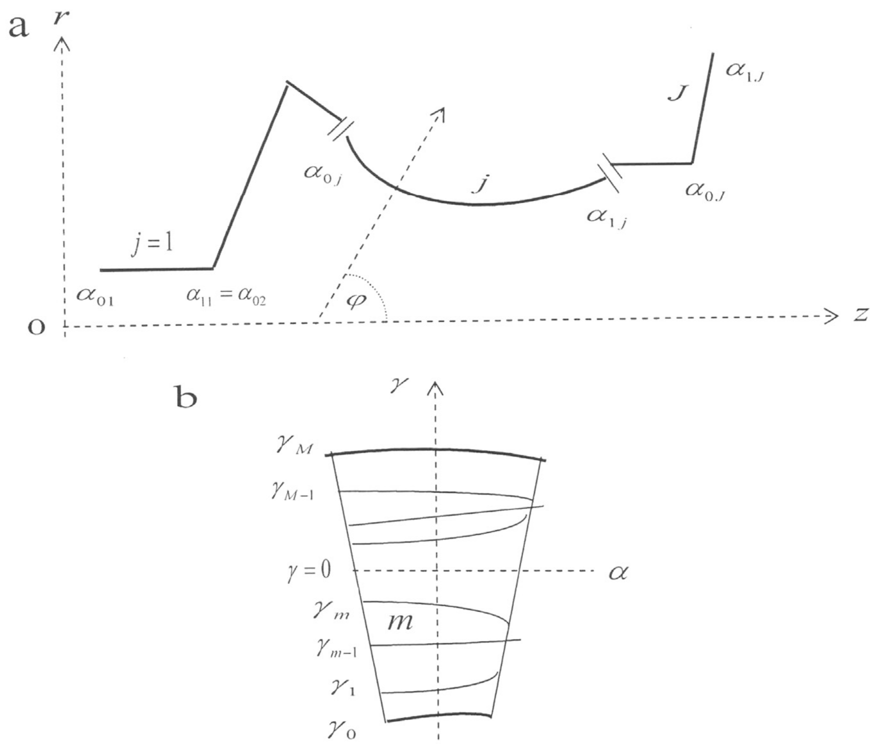

Fig. 1. The general view of the generatrix-meridian of the shell system (a) and its structure across the thickness (b).

hermetic capsule. In solving the problem, the trigonometric functions (circumferential direction) and Legendre's functions (longitudinal direction) were used.

The paper [12] addresses studying dynamical characteristics of joined together spherical and cylindrical shells under various boundary conditions. In calculations, the Flugge theory and Rayleigh-Ritz energetic method were used. The frequencies and vibration modes for the separate components of the system and for the system as a whole were compared with the data obtained with the FEM and (other available) analytical solutions. The Rayleigh-Ritz energetic method was involved also in other works, in particular, in solving the problem on free vibrations of joined together spherical and conical shells [13].

The modified variational approach, which was used in [10] in studying the cone-cylinder system, was employed in [14] for calculations of natural frequencies of a cone-cylinder-sphere system with ring stiffeners. Here the theoretical model is formulated within the framework of the Reissner-Nahdy theory for shells with reinforcing circumferential members. To approximate displacements of each member of the system, the orthogonal polynomials and trigonometric Fourier series were used. The ring stiffeners in shell combinations are treated as discrete elements.The Reissner-Nahdy theory was applied also in [15] to compute dynamical characteristics of a cylindrical shell with ends covered by semispheres using the domain decomposition method.

Besides, new engineering decisions of modern technology assume using of structures with sophisticated shapes that meet increased service requirements. This stimulates studying compound shells with members of complicated geometry, in particularly, with elliptical toroidal configuration (rockets, underwater vehicles, et al). Employment of such structures considerably extends the class of compound systems being studied including, as a particular case, the systems with zero and constant Gaussian curvature.

The present paper addresses dynamical characteristics of compound shells of revolution with elliptical toroidal members. The frequencies and modes of free vibrations of these systems are calculated within the framework of the Kirchhoff-Love classical model by using the numerical analytical technique developed. The technique assumes reducing an original two-dimensional eigenvalue problem, making use expansion into Fourier series, to the sequence of associated one-dimensional problems with the following solving of them for each series harmonic by the incremental search method in combination with the numerical orthogonal-sweep method.

The novelty of the given paper is governed by the following elements:

(i) class of the objects being considered (systems of conjugated shells of revolution with elliptical toroidal members);

(ii) complex numerical analytical approach for solving two-dimensional eigenvalue problems;

(iii) vibration peculiarities of a compound system established in comparison with vibrations of its members.

2. Problem statement and solving technique

Let us choose, as object for studying, the system consisting in a general case of J conjugated coaxial various-geometry shells of revolution. Each shell is considered as the separate j-th portion () of a single shell system whose coordinate (or middle, in a particular case) surface is formed by the rotation of a certain single-valued piecewise-smooth curve about the rectilinear $0z$-axis (Fig. 1(a)).

The location of the arbitrary point of this surface is determined in the orthogonal conjugate coordinate system (α, θ), where $\alpha = \{\alpha_j \in (\alpha_{0j}, \alpha_{1j})\}$ () varies along the meridian generatrix, and η is the central angle of the section $z = const$.The coordinate surface $\gamma = 0$ being the certain initial reference surface over the shell thickness is chosen informally, while the variable γ is referenced along the normal to this surface.

The shells may be single-layered across the thickness or be composed of M layers with constant or variable thickness along the meridian (Fig. 1(b)). The adjacent m-th and $(m + 1)$-th layers are in contact over the surface $\gamma = \gamma_m(\alpha)\,(m = \overline{1, M-1})$ without separation and slipping (conditions of perfect contact) while the external bounding surfaces $\gamma = \gamma_0(\alpha)$ and $\gamma = \gamma_J(\alpha)$ are free of loads. It is assumed that the material of layers is elastically deformed and may be isotropic or orthotropic, i.e with three planes of elastic symmetry that are orthogonal to the directions $\alpha = const,\ \theta = const,\ \gamma = const$.