

Full length article

Buckling solutions of clamped-pinned anisotropic laminated composite columns under axial compression using bifurcation approach and finite elements

Rund Al-Masri, Hayder A. Rasheed*

Department of Civil Engineering, Kansas State University, Manhattan, KS 66506, United States

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ABSTRACT

Following the bifurcation approach, a generalized closed form buckling solution for clamped-pinned anisotropic laminated composite columns under axial compression is developed using the energy method. The effective axial, coupling, and flexural rigidity coefficients of the anisotropic layups are determined following the generalized constitutive relationship using dimensional reduction by static condensation of 6×6 rigidity matrix. The presented analytical explicit formula reproduces Euler buckling expression in the case of isotropic or specially-orthotropic materials once the effective coupling term vanishes. On the other hand, the analytical formula furnishes two extra terms which are functions of the effective coupling, flexural and axial rigidity. The analytical buckling formula is confirmed against finite element Eigen value solutions for different anisotropic laminated layups yielding high accuracy for a wide range of stacking sequences. A parametric study is then conducted to examine the effect of ply orientations, material properties including hybrid carbon/glass fiber composites, FE element type, and column size. Relevance of the numerical and analytical results is discussed in comparison to previous results in literature for cross ply laminates.

1. Introduction

The distinguished properties of laminated composite material such as high stiffness-to-weight ratio, high strength-to-weight ratio, design versatility, as well as fatigue and corrosion resistance have captured the industry attention in the past few decades. Accordingly, the demand to understand the stability mechanics of laminated composite members has increased. Despite the fact that limited amount of research studies has addressed the buckling of anisotropic laminated composite columns, a significant amount of studies has been conducted on the stability of composite shells, plates, beams, and cylinders [1–15]. Al-Masri and Rasheed [1] presented a generalized analytical buckling formula of fixed-fixed anisotropic laminated composite columns using bifurcation approach. Rasheed Al-Masri stability equation is expressed in terms of the generally anisotropic material properties as well as the column geometry. The analytical results was confirmed with finite element analysis for various stacking sequences yielding a very good agreement with them. Moreover, the analytical formula was verified against previous existing solution for cross-ply laminated columns. Heidari-Rarani et al. [2] investigated the effect of angle-ply and cross-ply layups on the stability of E-glass/epoxy square composite laminated plates under

axial compression with SFSF (S: simply supported, F: Free) boundary conditions analytically, numerically, and experimentally. A semi-analytical solution was developed using Rayleigh-Ritz approach. Analytical results were verified against finite element analysis yielding an excellent accuracy. Moreover, Hashin, Tsai-Wu, and Tsai-Hill failure criteria were attempted in the numerical analysis to study the layer failure of the laminated composites. Experimentally, E-glass/epoxy plates of four layers were made with angle-ply ($[\pm 30]_s$ and $[\pm 45]_s$) and cross-ply ($[0/90]_s$) stacking sequences using hand layup method. The test was conducted under displacement control with a rate equal to 0.5 mm/min. On the other hand, the semi-analytical and numerical values overestimated the buckling loads compared to the experimental results. Tsai-Wu and Tsai-Hill failure criteria had the same failure mode as the tested plates in which the failure started from the plate edge then developed along the plate. Abramovich and Livshits [3] studied the free vibration of non-symmetric cross ply laminated composite beams based on the first order shear deformation theory. Longitudinal, transverse displacement, rotary inertia, and shear deformation were considered in the analysis. The new approach and Bernoulli-Euler theory were verified against numerical solutions. Debski et al. [4] studied the buckling and post-buckling of simply supported thin-walled composite channel

* Corresponding author.

E-mail address: hayder@ksu.edu (H.A. Rasheed).

column under axial compression loading experimentally. Carbon/epoxy columns of eight symmetrical plies $[0/-45/45/90]_s$ were tested using Zwick Z100/SN3A universal testing machine. The experimental results were verified with numerical (FE) results and analytical-numerical method (ANM) [5–8]. The $P-(\varepsilon_1 - \varepsilon_2)^2$ method underestimated the critical buckling load values in case of C-specimens with maximum error around 15.69%. Rasheed et al. [9] developed a closed form stability solution of simply supported anisotropic laminated composite plates under axial compression. They proved that Rayleigh-Ritz approximation produces an upper bound solution when compared to numerical results. A modified Rayleigh-Ritz formula, they derived, showed an excellent agreement with the finite element results for a wide range of stacking sequences. Furthermore, comparison with limited experiments was conducted showing a good correspondence. Silva et al. [10] presented a formulation of a generalized beam theory (GBT) to study local and global buckling behavior of fiber reinforced polymer (FRP) composite open section thin-walled columns. Silva et al. proved that Poisson ratio product $\nu_{12}\nu_{21}$ should not be neglected or else the buckling load results may be overestimated. Moreover, shear deformation had high effect on the flexural buckling results whenever the conventional deformation was assumed. Based on the classical lamination theory, Ahmadi and Rasheed [11] developed analytical solution to predict the lateral torsional buckling of anisotropic laminated rectangular composite beams under pure bending with simply supported boundary conditions. The analytical formula presented was confirmed against finite element analysis using software Abaqus yielding a very good correspondence. Furthermore, a parametric study was conducted to study the effect of various geometric and material parameters on the buckling load values. Cortinez and Piovan [12] presented a theoretical model to study the stability of composite thin-walled beams with shear deformability using nonlinear displacement field depending on Hellinger-Reissner principal. A finite element with fourteen degrees of freedom was used to solve the governing equations. The results showed that shear flexibility had a significant effect on the stability of the composite beams. Kubiak and Mania [13] studied buckling and post-buckling behavior of hybrid and fiber reinforced polymer (FRP) channel column section under uniform axial compression experimentally, and numerically. Channel column section of eight layers were made with various layer arrangement using fiber metal laminate (FML) and glass fiber reinforced polymer (GFRP). The results were confirmed against finite element analysis using software Ansys. The results showed that fiber metal laminate (FML) exhibited three times higher buckling load values than GFRP. Using first order shear deformation theory (FSDT) and FORTRAN, Akbulut et al. [14] studied the buckling behavior of angle and cross ply laminated composite stepped flat columns by developing a nine node finite element model (FEM) under compression load. Different boundary conditions, fillet angle, orientation angles, as well as column thickness and width were considered in estimating the buckling load values. The results showed that as the orientation angle increased, the composite column tends to buckle at lower load. Moreover, columns without fillet exhibited higher buckling load values than the ones with it. Rasheed et al. [15] proposed a generalized analytical lateral torsional buckling solution of simply supported anisotropic Steel-FRP thin walled rectangular beams under pure bending based on the classical lamination theory. The developed solution was confirmed against finite element analysis yielding high accuracy. Furthermore, a parametric study was performed to investigate the effect of different geometric and material parameters including the stacking sequence.

It is worth mentioning that the bifurcation approach that is followed in the present analysis is treated in the literature in three different ways. In approach I, the pre-buckling curvature κ_x is assumed to be zero resulting in an axial force and bending moment (due to the coupling of membrane deformations). Accordingly, the column will undergo fully straight pre-buckling deformation which was not observed in Fig. 4 below. Approach II assumes zero bending moment up to buckling which means that curvature κ_x exists in the pre-buckling solution when the

coupling coefficient is nonzero. This is what was observed in Fig. 4. Approach III attempts to transfer the centroidal axis to vanish the coupling terms. The authors believe that this cannot be applied in the very general case when the nonzero B matrix would not be completely zeroed out by simply shifting one centroidal axis. Accordingly, the authors followed the second approach.

The main goal of this work is to develop a generalized analytical buckling formula for anisotropic laminated composite clamped-pinned columns under axial compression loading using the bifurcation assumption for pre-buckling deformation. Applying the static condensation method, three dimensional 6×6 composite rigidity matrix is converted to one dimensional axial, coupling, and flexural rigidities. Moreover, the resulting formula has three terms as a function of the anisotropic rigidities and the analytical results were confirmed against finite element analysis using commercial software Abaqus yielding very good correspondence. Furthermore, comparison of the analytical results against previous findings for cross-ply laminates showed improved accuracy.

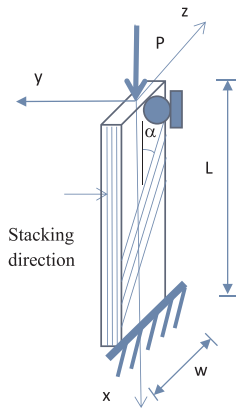
2. Analytical formulation

2.1. Assumptions and kinematics

Bifurcation solution is used to develop a generalized analytical critical buckling formula for clamped-pinned anisotropic laminated composite columns under axial compression. Prior to deriving the analytical solution, several assumptions are considered and presented in the following points:

- 1) Buckling takes place in the x-y plane about the weak axis (z-axis).
- 2) The y-axis runs through the thickness of the column where the composite lamination takes place, Fig. 1(a).
- 3) The lamination angle (α) is measured with respect to the x-axis (i.e. 0° fibers run parallel to the x-axis and 90° fibers run parallel to the z-axis). Accordingly, the angle (α) is rotated about the y-axis.
- 4) Plane sections before bending remain plane after bending and perpendicular to the mid surface (i.e. simple beam theory holds).
- 5) Classical lamination theory is applicable with shear deformations ignored.

Cartesian coordinates and geometry of the clamped-pinned columns used are illustrated in Fig. 1(a). The bending occurs about the z-axis (weak axis). The definition of each boundary conditions is presented in Fig. 1(b). The following displacement relations were assumed based on the isotropic Euler first buckling mode [16]:



(a)

Clamped support: $\begin{Bmatrix} u \\ \theta \end{Bmatrix} = \begin{Bmatrix} u_x \\ u_y \\ u_z \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$

Pinned support: $\begin{Bmatrix} u \\ \theta \end{Bmatrix} = \begin{Bmatrix} u_x \\ u_y \\ u_z \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} = \begin{Bmatrix} u_x \\ 0 \\ 0 \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}$

(b)

Fig. 1. The column geometry and boundary condition.

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