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Elastic buckling of thin-walled polyhedral pipe liners encased in a circular pipe under uniform external pressure

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ABSTRACT

In this study, a thin-walled polyhedral polymer pipe liner is proposed for the internal rehabilitation of a deteriorated/cracked underground circular metal pipe. The pipe liner is externally confined and subjected to the hydrostatic pressure of water seeped through the cracked pipe. The critical buckling pressure of the pipe liner is derived analytically based on the principle of minimum potential energy and compared with that of a cylindrical pipe liner. A finite element model of the pipe liner is established and analyzed to understand pressure-deformation equilibrium paths and the stability of post-buckling behavior. The analytical buckling pressure is in excellent agreement with the numerical results. The buckling pressure of a polyhedral liner increases with the increase of thickness-to-radius ratio and the decrease of the number of sides in polygon base shape. In comparison with the cylindrical liner, the polyhedral liner can increase buckling pressure up to 10 times but result in a less stable post-buckling behavior.

1. Introduction

In the past few decades, trenchless technologies with thin-walled polymer (steel) liners have been developed and employed to rehabilitate deteriorated pipelines due to their cost effectiveness and structural capacity [1–3]. Most of the existing pipes are structurally safe to support surrounding soils and surcharge loads but functionally obsolete when cracked and flooded. In this case, the pipe liners are only subjected to the hydrostatic pressure of water seeped through the cracked pipes surrounded by permeable mediums [4–6]. As the hydrostatic pressure builds up and exceeds a critical level, the liners may buckle and deform inward due to the confinement of the outer host pipes [7,8]. The buckled deformation shape of the liners likely appears in the form of "single symmetric inward lobe" [9] as illustrated in Fig. 1.

With the pre-determined deformation shape in Fig. 1, Glock [10] derived the buckling pressure of a thin-walled ring confined in a rigid cavity based on the principle of minimum potential energy. The friction between the ring and its surrounding cavity and the potential variation of the hoop compression force in the ring were neglected. The critical buckling pressure was simply expressed into:

$$\frac{P_{Glock}(1-\mu^2)}{E} = \left(\frac{t}{2R_0}\right)^{2.2} \tag{1}$$

where P_{Glock} is the critical buckling pressure, and E, μ, R_0 and t represent the Young's modulus, the Poisson's ratio, the mean radius and the wall thickness of the ring, respectively. Boot [11] modified the Glock's solution by taking into account the effect of initial gap between the ring and its surrounding cavity due to construction imperfections. Based on their experimental study, Aggarwal and Cooper [12] recommended an enhancement factor of 7 for the critical buckling pressure of a pipe liner due to its external confinement. Based on the finite element analysis of a pipe liner with various initial gaps from its host pipe, El-Sawy and Moore [13] proposed an empirical regression formula to predict the critical buckling pressure of the liner. The numerical result agrees well with the Glock's solution if the initial gap is set to zero.

Vasilikis and Karamanos [14] assumed a plastic collapse mechanism of a pipe liner as shown in Fig. 2 and examined a closed-form solution of the pressure that can be used to depict the post-buckling behavior of the pipe liner. Subsequently, El-Sawy and Sweedan [15] numerically investigated the effect of three-dimensional (3D) local imperfections on the buckling behavior of a cylindrical liner under external pressure, and suggested a new empirical formula that may aid designs in practice. Li

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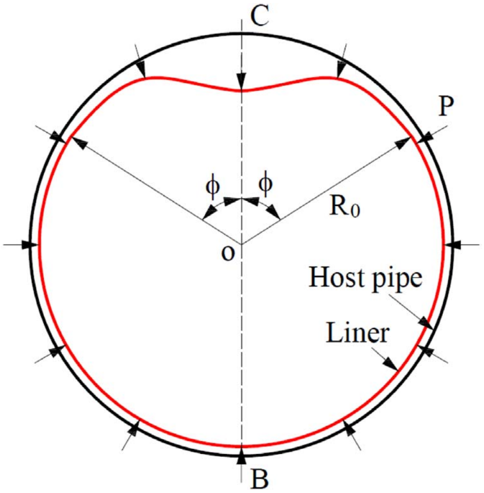


Fig. 1. Deformed shape of a liner under uniform pressure.

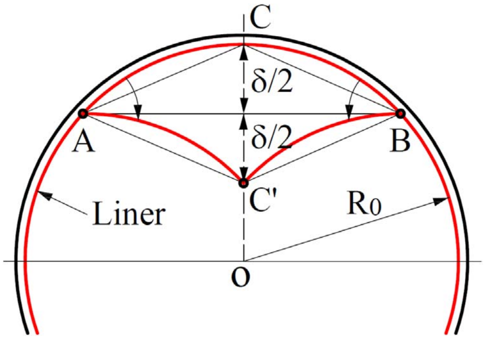


Fig. 2. The plastic collapse hinge model.

et al. [16] studied the buckling of a cylindrical pipe liner with varying thickness both analytically and numerically. The numerical results from a finite element model (FEM) were in good agreement with the analytical predictions.

In addition to elastic buckling of the liner subjected to external pressure, the elastoplastic material properties are also considered in many studies [17–22], all of which demonstrated that the liner with inelastic material behavior buckles at a lower pressure level than the elastic buckling pressure prediction as shown in Eq. (1). Boot [23] developed the creep buckling of thin-walled polymeric liners subjected to external pressure by introducing geometric nonlinearities and long term system behaviors. Rueda et al. [24,25] predicted the buckling pressure of a high density polyethylene (HDPE) liner accounting for the thermal effect and viscoplastic constitutive behavior by employing the FEM and experimental approach.

The above reviews indicate that the studies on confined liner buckling have been limited to circular liners encased in cylindric shells or rings. The shape of pipe liners has not been optimized in pipeline applications. Polenta et al. [26,27] exploited an innovative stiffener to improve the bending capacity of the liner, which shows a higher buckling load per unit mass than the one with a circular pipe. On the other hand, Miura [28] proposed a polyhedral shell to improve the post-buckling behavior of cans in beverage industry. Knapp [29,30] examined the stress distribution and stability of polyhedrical shells both experimentally and computationally. Knapp [31] also extended the application of polyhedral shells into undersea pressure hulls and numerically demonstrated that a polyhedral hull can resist higher pressure than its corresponding cylindric hull. After that, Albermani et al. [32] proposed the concept of a faceted cylindrical pipe that can increase the buckling capacity of a pipe liner without increasing its wall thickness based on the FEM analysis.

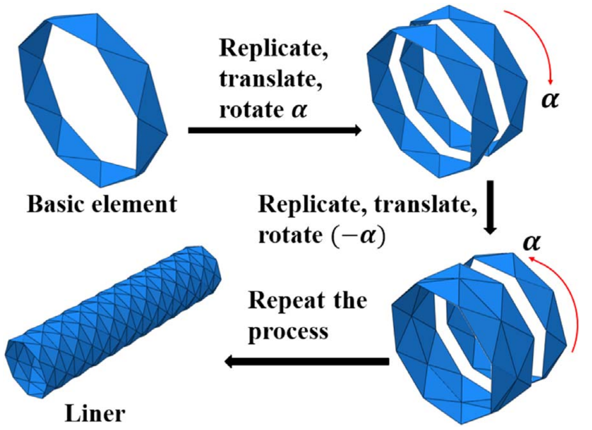


Fig. 3. Formation of a polyhedral liner.

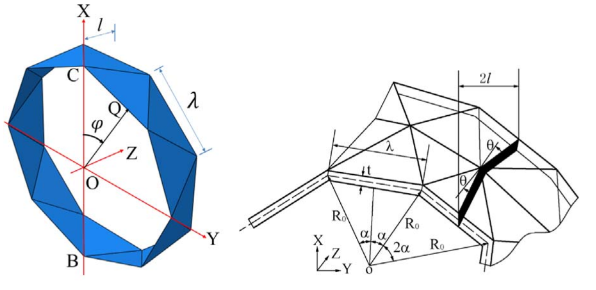
In this study, a thin-walled polyhedral pipe liner is proposed to rehabilitate underground circular pipes that deteriorate over time. The liner is assumed to be externally confined by a host pipe and subjected to uniform pressure. The critical buckling pressure of an elastic polyhedral pipe liner is first derived analytically based on the principle of minimum potential energy. The pipe liner is then simulated with a three-dimensional, geometrically nonlinear FEM [33] to investigate its post-buckling behavior and stability with large deformation and moving contact surface taken into account in addition to verifying the critical buckling pressure derived analytically. Finally, a shape factor of polyhedral pipe liner is defined as the ratio of the critical buckling pressures between the polyhedral liner and its corresponding circular liner. The effects of the number of surface triangles and the wall thickness of the polyhedral liner on the shape factor are investigated.

2. Geometric relations and potential energy

2.1. Formation and definition

A polyhedral liner can be formed by building, rotating and connecting basic elements along its axial direction. As delineated in Fig. 3, each basic element of N sides consists of $2N$ equilateral triangular faces placed next to each other with apexes alternated on two ends. Another basic element is connected to the first one after a clockwise rotation of $\alpha = \pi/N$ about the axis of the liner. A third basic element is connected to the second one after a counter-clockwise rotation of α about the axis of the liner, returning to the position of the first element. This process continues to complete the pipe liner. As such, the pipe liner is a periodic structure with a period of two adjacent basic elements in axial/longitudinal direction. In addition, any two adjacent elements are mirror imaged about their connection cross section.

When N is an even number, the two apexes in the opposite sides of a cross section, Points B and C in Fig. 4(a), are on a straight line passing through the center of the liner, Point O. A Cartesian O-XYZ coordinate



(a) Basic element (b) Geometry of element model

Fig. 4. Geometry parameters of the polyhedral liner.