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Static analysis of thin-walled space frame structures with arbitrary closed cross-sections using transfer matrix method

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A B S T R A C T

This paper develops a new transfer matrix method for the static analysis of thin-walled space frame structures with arbitrary closed single-cell sections, that is, thin-walled beam transfer matrix method (TBTMM), which expands the advantages of the general transfer matrix method (TMM). In this method, the bimoment B and the rate of twist of the beam $\varphi(x)$ are taken into account based on the Vlasov's beam theory and Kollbrunner-Hajdin assumption, and the bending and twisting can be calculated separately combined with the characteristic of Maillart's shear center. Using this method, the transfer matrix of single beam is obtained through Laplace transform and inverse Laplace transform at first, and then gets the coupling equations of the whole structure with respect to coordinate transformation matrices and joint coupling matrices. Some numerical examples are given and compared with the general TMM and the finite element method (B-FEM and S-FEM) results to validate the developed method.

1. Introduction

Thin-walled beams are extensively used in engineering structures, such as civil, mechanical and aerospace engineering. Due to the excellent stiffness-to-weight ratio, strength-to-weight ratio and stability, the thin-walled beams have been a topic of major interest among many researchers [1]. Timoshenko and Vlasov are the pioneers of thin-walled beam theory. Timoshenko [2] developed the bending and twisting theory of the thin-walled beam, while Vlasov [3] proposed to develop the general bending and twisting of thin-walled beam. Based on the assumptions of Vlasov's beam theory, Benscoter and Umansky [4,5] proposed and developed the theory of closed thin-walled beam. Maillart coined the concept of shear center and advocated the idea that the centroid is not the only important point in the cross-section, which provided the method for decoupling of bending and twisting. However, most of the researches for the thin-walled space frame structures were based on the open or symmetric cross-sections. Minghini [6] presented a simple and effective thin-walled beam model for buckling analysis of FRP structure. In Ref. [7] Bank et al. presented a modified beam theory for bending and twisting of open-section composite beams but warping is not considered in the theory. Vo and Lee [8,9] presented a flexural-torsional analysis of composite box beams in which the single beam system of the rectangular section was considered. Few attempts [10,11] have been made to analyze the problem of space frame structures with arbitrary cross-sections, that is, most researchers neglect the deformational influence of warping shear stress in relation to that of the Bredt's shear stress [12] and the coupling problem of bending and twisting. So it demonstrates good engineering signification to develop the thin-walled beam theory with arbitrary closed cross-section and apply it to the calculation of the space frame structure. The closed cross-section includes closed single-cell cross-section and closed multi-cell cross-section. The more general closed single-cell cross-section is used in this article. Yoo [13] studied the bimoment contribution to the stability of thin-walled assemblages. In Ref. [10], Tralli presented a hybrid finite element model for calculation of stresses and deformations. Chen and Blandford in Ref. [14] produced a C0 finite element formulation for thin-walled beams with the arbitrary cross-sections.

Thin-walled frame structures, the main kind of slender structures, are widely utilized to lighten engineering structures as well as save materials. For the complex thin-walled structures, their static and dynamic behaviors are highly important and those must be determined first. There are some commercial software (ANSYS, ABAQUS, SAP2000, ETABS et al.) that can do this. Most of them are based on the finite element method (FEM) [15,16], which can't establish the relationship

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between the cross-section's properties and the behaviors of the structure. But the objective of the current paper is to establish the relationship between the cross-section's properties and the structure's behaviors in the concept design stage without indeterminate cross-sectional shape.

There are a variety of numerical methods that have been developed to study the relationship between the cross-section's properties and the frame structure's behaviors except the FEM, such as the reverberation-ray matrix method (RRMM) [17,18], the transfer matrix method (TMM) [19–22], and the stiffness matrix method [23–26]. Among them, the TMM can get the exact solution and is used to reduce the matrix sizes. In Ref. [27–31], TMM is exceptionally suitable for large systems that are made up of several subsystems. However, only a handful of references in published literature refer to the application of thin-walled beams [16] and the calculation of space frame structures [20,28].

Another objective of the present paper is to develop the transfer matrix method for analyzing the static behaviors of space frame structures with arbitrary closed single-cell cross-sections, which as a new method of analysis is called thin-walled beam transfer matrix method (TBTMM), and two specific examples were considered to illustrate the method. First, static behaviors of the space frame structure with arbitrary closed single-cell cross-sections were computed by MATLAB based on TBTMM. The obtained results were compared with those of the general TMM and the finite element method by using ANSYS with beam elements (beam188) and shell elements (shell63).

2. Basics of the theoretical and mathematical model

2.1. Kinematics assumptions and displacement field of thin-walled beam

The element geometry of single beam is shown in Fig. 1a, in which S-point is the shear center of the cross-section and the C-point is the centroid of the cross-section. In order to describe the displacement field of the beam cross-section, two coordinate systems are adopted: the right-hand orthogonal coordinate system (xyz) and the s-coordinate system which is along the midline of the cross-section. The yz-plane of the right-hand orthogonal coordinate system is parallel to the cross-section, and the x-axis coincides with the beam axis passing through the centroid C of each cross-section, and the y- and z-axes are the principal inertia axes of the cross-section. $h(s)$ is the perpendicular distance from the shear center to the s-axis and the thickness t of the cross-section is a function of s in the x-direction as shown in Fig. 1b. The transverse

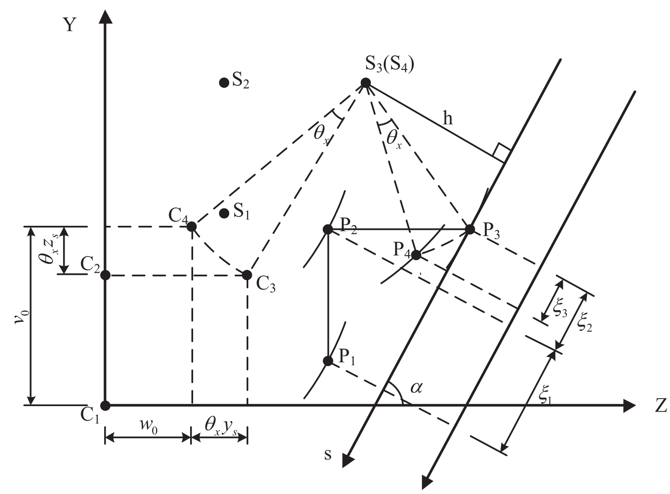

Fig. 2. Transverse displacement of the point P along the midline of the cross-section.

displacement of an arbitrary point P in the midline of the cross-section is shown in Fig. 2. C_i ($i = 1, 2, 3, 4$) and S_j ($j = 1, 2, 3, 4$) represent the displacement processes of the centroid and the shear center, respectively. $u_0(x)$, $v_0(x)$ and $w_0(x)$ are the rigid-body translations of the cross-section associated with the centroid in the x-, y- and z-directions, respectively; θ_x is the angle of twist about the shear center in the x-direction; θ_y and θ_z are the rigid-body rotations about the y- and z-axes, respectively; α is the angle between the z-axis and the tangential direction at point P; and ξ is the tangential displacement at the point P. In general, for the coupling problem of bending and twisting of the thin-walled beam, we need to consider the bending in the xy- and xz-planes, the twisting about the shear center in the x-direction, and the axial tensile and compression. But as Maillart coined the concept of shear center, the bending and twisting could be calculated separately.

The kinematic behaviors of arbitrary closed single-cell cross-section beam can be derived by using the following assumptions [13,32]

(1) The Vlasov's beam theory is valid for twisting of the thin-walled beam, that is, the cross-section is not deformed in its plane but is subjected to warping in the longitudinal direction.
(2) The deformations are small with respect to the dimensions of the cross-section (linearized problem).

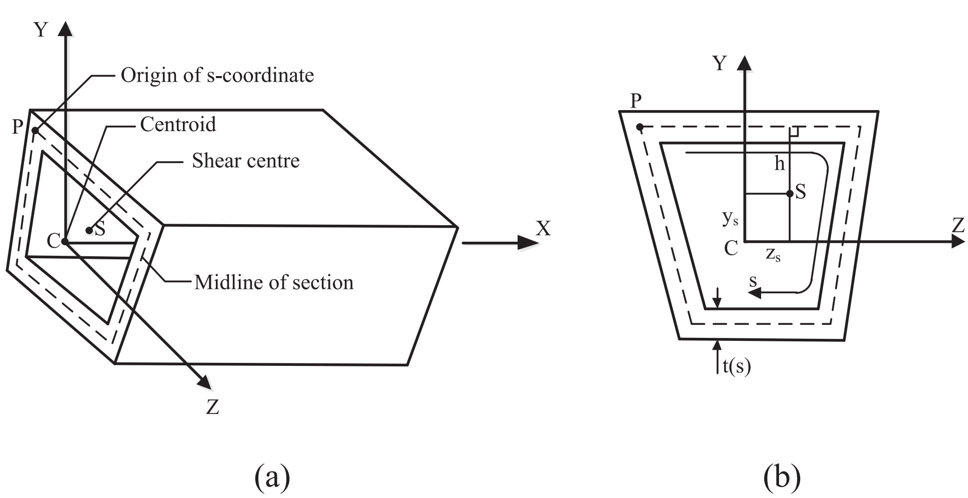

Fig. 1. Element geometry of a single beam (a) and the details of the cross-section (b).