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A finite strip for the vibration analysis of rotating cylindrical shells

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ABSTRACT

In this article, a two-node finite strip with eight degrees of freedom for the free vibration analysis of pre-stressed rotating cylindrical shells is formulated. The circumferential mode shape profiles are described exactly using trigonometric functions. The axial mode shape profiles are approximated by bar and beam shape functions for membrane and bending displacements, respectively. In this way, a semi-analytical formulation is facilitated so that discretisation is required only in the axial direction. The accuracy and convergence of the developed finite strip are confirmed by comparisons with the analytical results. Excellent agreement is observed both for stationary and rotating shells.

1. Introduction

Shells are widely used as constructive elements in many engineering structures. Their static and dynamic behaviour has been an important topic in structural design for a long time [1–3]. As a result, the theory of shells and plates has been covered in a systematic manner in a number of books [4–11]. For example, an instructive approach to thin shell theory, written in a relatively simple way and adapted to the engineering level for practical usage, is presented in [12]. On the other hand, some authors have covered more specific problems related to the design of shell-like structures [13,14]. For example, the general theory and specific discussions regarding shells of revolution exposed to a uniform load can be found in [15]. Such problems are more particular for submarine or aircraft pressure hull designs with pronounced axial symmetry.

In fact, it is often the case that axisymmetric shells rotate around the axis of symmetry [16–22,24–30]. Rotating shells of revolution are found in engineering practice in rotor systems of gas turbine engines, high-speed centrifugal separators, rotating satellite structures, automotive tyres, etc. Rotation makes their dynamic behaviour significantly more complex. One of the first investigations into the vibration of rotating structures was carried out by Bryan [16]. He studied the vibrations of a rotating ring and described the travelling modes phenomenon. These phenomena result from the Coriolis effect, as shown in the example of infinitely long rotating cylindrical shells [17,18], as well as in finite rotating cylinders [19,20]. An experimental study on the flexural vibrations of a thin rotating ring is given in [21]. Furthermore, the influence of pre-stress on the free vibrations of rotating cylinders has been studied in [22].

Huang and Soedel [24] used nonlinear strain-displacement relationships [23] to formulate the corresponding set of differential equations of motion for a rotating cylindrical shell. They exactly solved the free and forced vibration problem of a simply supported cylindrical shell by assuming simple sine and cosine displacement functions of the circumferential and axial variables. In this case, formulating the eigenvalue/eigenvector problem results in a characteristic polynomial of the sixth order. Its solution gives three positive and three negative natural frequencies.

If the shell does not rotate, the polynomial is bicubic. There are three pairs of positive/negative frequencies characterised by the same absolute value. This is physically explained through pairs of backward and forward rotating modes. The two modes of a pair rotate with the same speed in opposite directions and thus superimpose into a stationary mode. The reason why there are three pairs of modes and natural frequencies is that there are three types of dominant modes: bending (radial), longitudinal (axial), and shear (circumferential).

If the shell rotates around its axis with a constant speed, then the polynomial is no longer bicubic. The full sixth order polynomial occurs. The positive and negative natural frequencies have distinct absolute values and so-called frequency veering (bifurcation) happens. This means that the forward and backward rotating modes no longer rotate with the same rotation speed and thus cannot superimpose into the stationary modes (standing waves). As a result, with spinning shells the modes rotate independently. For example, this phenomenon comes about with tyres rolling over the road surface [25–29]. This significantly influences the overall NVH (Noise and Vibration Harshness) characteristics of the vehicle.

It is very common in the literature on the vibration of cylindrical

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shells, either rotating or stationary, to assume that the two ends of the shell are simply supported [24,30,31]. This type of boundary support is sometimes referred to as a shear diaphragm set of boundary conditions [11,12]. The reason why this set of boundary conditions is often used is that it is probably the only one for which a relatively simple solution can be obtained analytically. In other words, mode shapes assumed as appropriate products of trigonometric functions of the circumferential and axial variables usually satisfy both the differential equations and the boundary conditions. This results in a mathematically convenient model.

However, such a model is not necessarily suitable to describe a particular engineering problem. For example, although it is very instructive to investigate the dynamics of rotating tyres assuming simply supported edges of a tyre tread-band [24,30,31], it is difficult to accept, from an engineering point of view, that the tyre sidewall is infinitely stiff in radial or tangential directions [28,29].

In the case of cylindrical shells with boundary conditions other than simply supported ones, the mathematics becomes significantly complicated. Closed-form solutions are now difficult to obtain. A number of investigations have been undertaken to tackle this problem [32–38]. One of the solutions was obtained by assuming the shell displacement field as a product of Fourier series in the axial direction, and trigonometric functions in the circumferential direction [35]. This procedure has been recently extended to rotating cylindrical shells [36]. The problem of the free vibration of a rotating cylindrical shell having arbitrary boundary conditions can also be solved by employing the Rayleigh-Ritz method. Such a solution, using characteristic orthogonal polynomials for displacement variations along the axial direction, can be found in [37].

A complete analytical solution for free vibrations of a rotating cylindrical shell with arbitrary boundary conditions has recently been offered in [38]. The equations of motion in [38] are based on the strain-displacement relationships of Hermann and Armenakas [23]. Circumferential tension due to internal pressure or centrifugal forces, as well as an elastic foundation in both radial and circumferential directions, is taken into account. The circumferential mode shape profiles are described by trigonometric functions. The axial profiles are assumed as a sum of eight weighted exponential functions. Three differential equations of motion for an assumed circumferential mode number lead to a frequency equation in the form of a bi-quartic polynomial. Eight cases of the four different combinations of the polynomial roots (real, imaginary and complex) were identified. Hence, the mode shape axial profiles are described in terms of trigonometric functions, hyperbolic functions and their products. The application of the analytical solution was illustrated in the case of a cylindrical shell with free-free boundary conditions, and excellent agreement with the experimentally obtained results was confirmed. The principal advantage of the analytical procedure [38] is very high accuracy confirmed by experiments.

However, the procedure requires the discovery of a proper case among the eight types of axial mode shapes for which a solution can be found even for a single cylindrical shell. If a shell structure consists of n cylindrical shells of different particulars, the number of combinations to find the proper one is 8^n. Hence, although numerical examples solved in this way may be very useful as benchmarks for the evaluation of various numerical methods, the analytical procedure is not quite suitable for practical use.

In practical situations, the problem of the vibration of cylindrical shells with boundary conditions other than simply supported ones could be solved by using the finite element method (FEM). In fact, complex built-up shell structures, which may be approximated by a number of connected cylindrical or other types of shells, could be conveniently tackled by the finite element method. For this purpose, special shell finite elements based on the waveguide finite element method would be suitable. Such finite element analyses have so far been used to tackle stationary shell structures [42,43]. Alternatively, the homogeneity of the cylinder around the circumference and along the axis has been exploited to post-process the FE model of a small rectangular segment of the cylinder using periodic structure theory to obtain the wave characteristics of a cylinder [44]. Since there is an integer number of wavelengths around the circumference of a closed circular cylinder, one of the integrals in the inverse Fourier transform becomes a simple summation, whereas the other can be resolved analytically using contour integration and the residue theorem [44].

Although the semi-analytical waveguide approaches of [42–44] may be very convenient and useful for a variety of problems, none of the semi-analytical finite element formulations developed so far a) allow for considering the typical effects of rotation (rotating modes and frequency veering) or b) can take into account the effects of pre-stress due to possible pressurisation and/or centrifugal forces. Therefore, the state-of-the-art in the considered field motivates further developments of finite elements especially tailored to model rotating and pre-stressed cylindrical shells.

One of the very effective numerical methods which reduces the two-dimensional problem into a one-dimensional one in the case of simply supported two opposite edges or shells of revolution is the finite strip method (FSM) [45]. Due to this advantage, the method is widely used in the structural analysis of engineering structures. Some recent publications on different problems are included in the reference list [46–54]. All the articles are published in the Thin-Walled Structures journal as a major forum for the development of the finite strip method.

In this paper, a new finite strip for modelling pre-stressed rotating cylindrical shells is formulated. The energy approach is used with the strain-displacement relationships given in [23,24] to develop and validate such a special finite strip [45]. The strip element is deliberately made quite simple, including two nodes and eight degrees of freedom (d.o.f.). Its reliability is checked in a number of numerical examples by comparing the numerical results with the analytical ones. Excellent agreement is observed for all the boundary conditions considered.

2. Strain and kinetic energy of a rotating cylindrical shell

A thin cylindrical shell rotating around its axis of symmetry with constant angular speed Ω is shown schematically in Fig. 1. The shell dimensions are the following: L is the length, a is the radius, and h is the thickness. The shell mid-surface is defined in the cylindrical coordinate system, where x and ϕ are the axial and angular coordinates, respectively. The displacement of a point P on the mid-surface, whose position is defined by x and ϕ, is specified by the axial, tangential and radial displacement components u, v and w, respectively, as shown in Fig. 1.

The problem of shell vibration can be analysed by directly solving differential equations of motion or by minimising the total energy with assumed mode shapes, i.e. by applying the Rayleigh-Ritz method. The equations of motion can be derived by considering the equilibrium of the internal and external forces on an infinitesimally small element [12]. Differential equations of motion can also be obtained by minimisation of the total energy with respect to displacements, i.e. by applying Hamilton's principle.

In the case of a rotating cylindrical shell, the latter approach is more convenient, as shown for example in [37]. Love's simplification [55] allows for the shell strain field to be separated into membrane strains and bending strains [24]

$$\tilde{\varepsilon}_x = \varepsilon_x + z\kappa_x, \quad \tilde{\varepsilon}_\phi = \varepsilon_\phi + z\kappa_\phi \quad \tilde{\varepsilon}_{x\phi} = \varepsilon_{x\phi} + z\kappa_{x\phi}, \tag{1}$$

where ε_i are the membrane strains, κ_i denote the change in curvature due to the shell bending, and z is the distance of a shell layer from the