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Optimization of in-plane functionally graded panels for buckling strength: Unstiffened, stiffened panels, and panels with cutouts

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ABSTRACT

The work of this paper deals with the in-plane material optimization with the objective of minimizing the amount of the nano-reinforcement required to satisfy the desired buckling constraints. The minimization of the reinforcement is necessary for nano-reinforced composites because the price of the reinforcement is very high. Three types of panels are considered; (1) unstiffened panel, (2) panels with cutouts, and (3) stiffened panels. The in-plane distribution of the reinforcement is represented using the polynomial expansion technique which is also extended to model non-rectangular domains via coordinates transformation. It was found that material grading can saves a very significant amount of the reinforcement up to 200% relative to homogenous panels. The saving of the reinforcement depends on four factors; (1) the problem nature, (2) the boundary conditions, (3) the applied loads, (4) the direction of the material gradings.

1. Introduction

A Functionally graded structure (FGS) is a structure which has a continuous variation of the material properties in one or more of its dimensions. This concept was first proposed in Japan in 1987 during the design of the thermal shield of spacecraft [1–3]. The shield should have a gradual variation of the thermal expansion coefficients to sustain the high heat on the outer side and at the same time to be consistent with the thermal expansion of the internal structure to avoid the stress concentration at the interface. Since that time much progress has been made in the design, analysis, and the manufacturing of functionally graded structures. A comprehensive review of the progress in the analysis and design can be found in [4–9]. Regarding the manufacturing of FGS, the available technology allows the production of very complex shapes with material grading in multiple directions. Some of these techniques are the innovative thermal spraying technique which sprays molten materials onto a surface. This Technique allows the process of very fine particles in the range of the nanometer. Another technique is the Laser Engineered Net Shaping (LENS) which allow the manufacturing of 3D parts by injecting metal powder into a molten medium created by a high-powered laser beam. More of the manufacturing techniques can be found in [10–14].

The material gradation throughout the structure can be represented mathematically by numerous ways. This first method is the power-law representation in which the volume fractions of the compositions are represented by monomials with two design parameters; the monomial's coefficient, and the power. The second method is the exponential representation where the volume fraction is represented by an exponential function with also two design parameters; the coefficient and the exponential function argument which is usually a linear function of the dimension in which the grading is desired. It can be seen that these two methods have one disadvantage which is their monotonic nature, so the volume fraction is either a decreasing or an increasing function which somehow hinders the design flexibility. Aragh et al. [15] modified the power-law by adding a linear term to the traditional power-law and an overall power to increase the design parameters to three. Still the variation nature is limited and attention should be paid to the parameters values because the volume fraction can exceed the permissible range which is between *0* and *1*, so the optimization process can be tricky. The third method is the grid-based representation in which a grid is created over the structure, and the volume fraction at each node (control point) is considered as a design variable, then any interpolation technique like the B-spline techniques can be used to get the global distribution [16]. This method provides much more flexibility compared to the first two methods, but requires more design variables which makes it computationally expensive. The fourth method proposed in [17] is the polynomial expansion of the volume fraction where the expansion's coefficients are the design variables. The volume fraction is maintained within the permissible limits over the structure domain via sets of linear and non-linear constraints. This method provides flexible designs with a low number of design variables which makes it a link between the power-based methods and the grid-

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based methods, so it will be adopted here to study the mechanical buckling of functionally graded panels.

Some of the work done in the literature regarding this topic is the work of Feldman and Aboudi [18] where the distribution of the Silicon Carbide volume fraction in an Aluminum matrix was optimized to maximize the buckling strength. The volume fraction was represented by a series expansion resulted from the tensor product of the Legendre polynomials in three dimensions. The only mentioned constraint was that the overall volume fraction should not exceed a certain limit. The used optimization technique, the constraints on the expansion coefficients, nor the number of terms used in the expansion have not been mentioned though.

Chu et al. [19] also considered the buckling of plates with in-plane inhomogeneity where the material grading is represented by the power and exponential laws. Lal and Saini [20] studied the buckling of simply supported plates under linearly varying in-plane loads with exponentially graded material properties along the direction perpendicular to the loads. Bodaghi and Saidi [21] studied the buckling of plates resting on elastic supports with nonlinear in-plane loading, and material grading throughout the plate's thickness for different types of boundary conditions.

Nowadays, there is a keen interest in the design of nanocomposite structure because of their superiority over the traditional fiber or microcomposites in terms of the stiffness and strength. Many research articles have been published regarding the bucking of composite plates and shells reinforced by carbon nanotubes [22–30]. Most of that work considered the material grading throughout the thickness using the linear variation or via the power-law. The cost of the nano reinforcement is very high [31] compared to micro or long fiber reinforcements which necessitates a structure to be designed with minimum reinforcement. So, the objective of this work is to optimize the in-plane distribution of the reinforcement to satisfy certain buckling strength constraints. Silicon Carbide particles-reinforced Aluminum matrix is considered to study the buckling of unstiffened, stiffened panels, and panels with cutouts. The volume fraction of the Silicon Carbide is presented using the polynomial expansion approach which will be extended here to optimize complex non-rectangular domain via domain transformation.

2. The theoretical model

2.1. The polynomial expansion of the volume fraction

The polynomial expansion representation as given in [17] is based on the decomposition of the volume fraction into two or more polynomials (components) in terms of non-dimensional coordinates ($0 \le x^*, y^* \le 1$) as follows:

$$v_f(x^*, y^*) = v_{fx}(x^*) v_{fy}(y^*) \tag{1}$$

$$v_{fx}(x^*) = \sum_{i=0}^{n} \gamma_i \, x^{*i}, \ and \ v_{fy}(y^*) = \sum_{j=0}^{m} \alpha_j \, y^{*j} \tag{2}$$

The polynomial expansion coefficients α_i, and γ_i are the design variables to be obtained through the optimization process. The sets of constraints on the coefficients required to maintain the value of the volume fraction within the permissible limits ($0 \le v_f(x^*, y^*) \le 1$) throughout the structure domain $\Omega_{(x^*, y^*)}$, can be developed as given in [17] by considering the values of the volume fraction components at $x^* = 1$, and $y^* = 1$ which give:

$$0 \le \sum_{i=0}^{n} \gamma_i \le 1, \ \ 0 \le \sum_{j=0}^{m} \alpha_j \le 1 \tag{3}$$

The integration of the volume fraction components over the structure domain $\Omega_{0 \le (x^*, y^*) \le 1}$ yield:

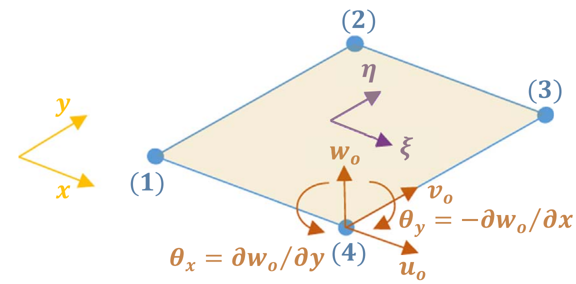

Fig. 1. Four nodded quadrilateral element with five degrees of freedom per node. (ξ, η) are the element's local axes, while (x, y) are the global axes.

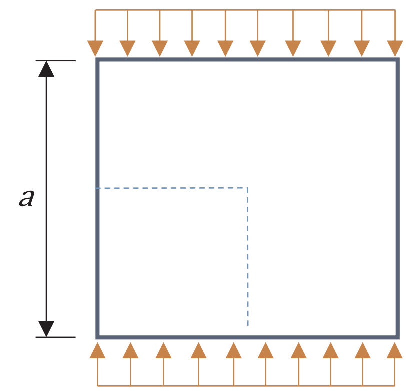

Fig. 2. Square panel under in-plane uniaxial loading.

Table 1
The optimal Silicon Carbide distribution for different expansion orders for the simply supported panel under uniaxial in-plane loading.

Expansion order	$v_f(x^*)$	V_{fSiC}
Constant	0.4234	0.1793
Linear	$0.8222x^*$	0.1690
Quadratic	$0.0795+x^*-0.5x^{*2}$	0.1704
Cubic	$0.8726x^*+0.2763x^{*2}-0.4751x^{*3}$	0.1678
Quartic	$0.8311x^*+0.4943x^{*2}-0.7971x^{*3}+0.1429x^{*4}$	01678

$$0 \le \sum_{i=0}^{n} \frac{\gamma_i}{i+1} \le 1, \ 0 \le \sum_{j=0}^{m} \frac{\alpha_j}{j+1} \le 1 \tag{4}$$

This constraint set imposes that the overall Silicon Carbide volume fraction $V_{fSiC} = \iint v_f \, dx^* dy^*$ is less than unity, it also improves the performance of the optimization process. If C[1] continuity is desired across the symmetry lines ($x = \bar{x}^*$or $y = \bar{y}^*$ if existed), then the first derivatives give:

$$\sum_{i=1}^{n} i\gamma_i \, \bar{x}^{*i-1} = 0, \ \sum_{j=1}^{m} j\alpha_j \, \bar{y}^{*j-1} = 0 \tag{5}$$

This set can also be solved to get the maxima and minima of the volume fraction within the structure domain to ensure that it does not violate the permissible limits. The range of each coefficient is as follows:

$$0 \le (\alpha_o, \gamma_o) \le 1, \ -1 \le (\alpha_j, \gamma_i) \le 1 \tag{6}$$

The range of $\alpha_o$and γ_o are obtained by considering the value of the volume fraction at the origin, while the range of $\alpha_j$andγ_i is optional. Faster convergence is obtained for the given range in Eq. (6) specially if heuristic approaches are used for optimization.