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Bending of thin rectangular plates with variable-thickness in a hygrothermal environment

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ABSTRACT

This paper presents hygrothermomechanical bending analysis of variable-thickness thin rectangular plates. Two opposite edges of the present variable-thickness plate are clamped while the other opposite edges are simply-supported. The present plate has varying thickness between its two edges and found in a hygrothermal environment. The exact analytical solutions are developed for the bending behavior of thin rectangular plates subjected to transverse uniformly distributed load. Numerical results are presented by using both Lévy-type approach and the small parameter method. A validation example is employed with existing literature results. More reported and illustrated data for deflections, bending moments and bending stresses are investigated.

1. Introduction

Variable thickness structures are extensively used in many types of high-performance surface like aircrafts, civil structures and other engineering applications [1–8]. The use of variable-thickness structural elements helps to reduce the weight of structural elements. This leads to improve the utilization of the materials to more economical design of structures. Kashtalyan and Nemish [9] considered a 3D boundary-value problem of bending analysis of an orthotropic plate with thickness variation and nonplanar faces perpendicular to the load. Shufrin and Eisenberger [10] presented accurate numerical techniques for vibration frequencies of variable-thickness rectangular plates with various combinations of boundary conditions. Xu and Zhou [11,12] presented the three-dimensionally elasticity solution for rectangular plates and continuously varying thickness functionally graded plates simply-supported at four edges. Vivio and Vullo [13] presented closed form solutions of axisymmetric bending of circular plates having non-linear variable thickness according to a power of a linear function. Semnani et al. [14] used Hamilton's principle to treat the vibration frequencies of variable-thickness thin plates. Yuan and Chen [15] presented an exact analytical method to solve axisymmetric flexural free vibrations of radially variable-thickness inhomogeneous circular Mindlin's plates. A literature review for variable-thickness plates shows that the analyses have done based on different methodologies [16–23].

With the ever-increasing applications of composite structures in severe environmental conditions hygrothermal response of such structures has attracted enormous awareness. The structural constituents of high-speed aircrafts, space crafts and re-entry space vehicle encounter hygrothermal loading conditions [24–26]. During the service period of variable thickness structures, they may be exposed to hot and/or humid environmental conditions, in such conditions the strength and stiffness of composite degrades; because of moisture diffusion, thermal spikes and thermal oxidation. York [27] presented new tapered hygro-thermally curvature-stable laminate configurations for which mechanical coupling and immunity to thermal distortion are maintained. Gayen and Roy [28] dealt with an analytical method to determine the stress distributions in laminated circular tapered composite beams under hygrothermal loadings. Mashat and Zenkour [29] presented hygrothermal bending behavior of a sector-shaped annular plate with variable radial thickness based on the classical plate theory.

Small parameter method (SPM) is an active numerical method for the solution of different differential equations to derive solutions in polynomial form. In recent publications, many researchers have successfully utilized SPM to solve many kinds of linear or non-linear one-dimensional problems of plate with variable thickness. In this article, an analytical solution based on Levy-type solution and a numerical approach based on the small parameter method is presented for prediction of the effect of transverse deflection and stresses of the rectangular plate. The thickness variations and the hygrothermal response will be investigated. The effect of thickness variation and hygrothermal environment on the bending response of the rectangular plates is discussed. The results of stresses and deflections are compared with the corresponding ones available in the literature. Additional results are presented to serve as benchmarks for future comparisons with other investigators.

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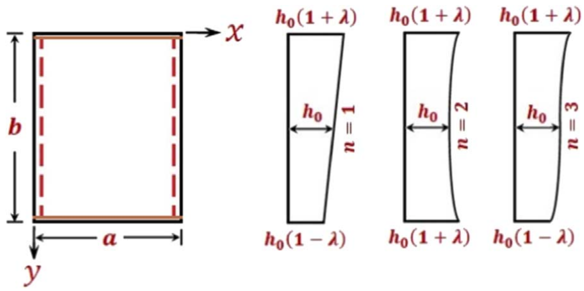

Fig. 1. Geometry and coordinate system of the variable-thickness rectangular plate.

2. Formulation of the problem

Consider a supported-clamped rectangular plate with variable-thickness h as shown in Fig. 1. For the analysis of such plate, we use a Cartesian coordinate system $Oxyz$ with the origin O located at the mid-plane of the plate. The present variable-thickness rectangular plate is subjected to moisture exposure $C(x, y, z)$ and elevated temperature $T(x, y, z)$. Moreover, the upper surface of the present plate is subjected to a uniformly distributed load $q(x, y)$. The thickness of rectangular plate varies along y direction, that is, $h(y)$ with reference thickness h_0. The thickness-ratio $h(y)/h_0$ is assumed to be everywhere appropriately small to stratify the classical thin plate theory for plates with thickness variations.

The flexural moment M_x, M_y and M_{xy} for the plane stress assumption take the form

$$\{M_x, M_y, M_{xy}\} = \int_{-h/2}^{+h/2} z\{\sigma_x, \sigma_y, \tau_{xy}\}\mathrm{d}z, \tag{1}$$

where σ_x, σ_y and τ_{xy} are the stress components

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} - \frac{E}{1-\nu}\begin{Bmatrix}1\\ 1\\ 0\end{Bmatrix}(\alpha T + \beta C), \tag{2}$$

in which α represents thermal expansion coefficient and β represents moisture concentration coefficient of the rectangular plate, E represents Young's modulus, ν denotes Poisson's ratio and ε_x, ε_y and γ_{xy} represent the following strain components:

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \quad \gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}. \tag{3}$$

The substitution of Eq. (2) in Eq. (1), with the help of Eq. (3), gives

$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix} = -D\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & 1-\nu\end{bmatrix}\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ \frac{\partial^2 w}{\partial x \partial y}\end{Bmatrix} - \begin{Bmatrix}1\\ 1\\ 0\end{Bmatrix}M^{TC}, \tag{4}$$

where D is the plate flexural rigidity and $M^{TC} = M^T + M^C$, in which the hygrothermal resultants M^T and M^C are given by

$$\{M^T, M^C\} = \frac{E}{1-\nu}\int_{-h/2}^{+h/2} z\{\alpha T, \beta C\}\mathrm{d}z. \tag{5}$$

The governing differential equation in terms of plate deflection w should be given by

$$D\nabla^2\nabla^2 w = q(x, y) - 2\frac{\partial D}{\partial x}\frac{\partial}{\partial x}(\nabla^2 w) - 2\frac{\partial D}{\partial y}\frac{\partial}{\partial y}(\nabla^2 w) - \frac{\partial^2 D}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - \frac{\partial^2 D}{\partial y^2}\left(\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - 2(1-\nu)\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} - \nabla^2 M^{TC}, \tag{6}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator.

It is supposed that the present rectangular plate has non-uniform thickness h along y direction as shown in Fig. 1. One can get at any point of the plate the non-uniform thickness as

$$h(y) = h_0[1+\lambda f_n(y)], \tag{7}$$

where λ represents a small parameter, n is the degree of thickness variability of the function $f_n(y)$ that characterizes the thickness variation of the plate and h_0 represents the reference thickness of the plate. The values of λ can be selected in a small range so that the values $f_n(y)$ should be less than or equals unity. This is keeping the plate thin enough to fall within the range of classical plate theory. The function $f_n(y)$ may be assumed in the form

$$f_n(y) = \left(\frac{2y}{b} - 1\right)^n, \quad n = 1, 2, 3, ... \tag{8}$$

This formula gives different variable thickness of the present plate, i.e., the tapered or linear thickness variation ($n = 1$), the quadratic thickness variation ($n = 2$), and the cubic thickness variation ($n = 3$) as shown in Fig. 1. Using Eq. (7), the plate flexural rigidity D will be

$$D = D_0[1+\lambda f_n(y)]^3, \quad D_0 = \frac{Eh_0^3}{12(1-\nu^2)}, \tag{9}$$

where D_0 is the constant flexural rigidity. According to Eqs. (7)–(9), Eq. (6) tends to

$$\nabla^4 w = \frac{1}{D_0}[1+\lambda f_n(y)]^{-3}(q - \nabla^2 M^{TC}) - 6\lambda^2[1+\lambda f_n(y)]^{-2}\left(\frac{\mathrm{d}f_n}{\mathrm{d}y}\right)^2\left(\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - 3\lambda[1+\lambda f_n(y)]^{-1}\left[2\frac{\mathrm{d}f_n}{\mathrm{d}y}\frac{\partial}{\partial y}(\nabla^2 w) + \frac{\mathrm{d}^2 f_n}{\mathrm{d}y^2}\left(\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right]. \tag{10}$$

The general elevated temperature $T(x, y, z)$ and moisture exposure $C(x, y, z)$ are expanded as

$$\{T, C\}(x, y, z) = \frac{1}{\mu^2[h(y)]^2}\{\check{T}(z), \check{C}(z)\}\sin\mu x, \tag{11}$$

where $\mu = j\pi/a$, $\check{T}$ represents the distribution of temperature through-the-thickness of the rectangular plate and $\check{C}$ denotes the distribution of moisture concentration through-the-thickness of the rectangular plate. The above form is chosen such that the Laplacian of the hygrothermal terms should be appeared as sinusoidal functions of x only. The temperature distribution may be described without heat generation function and the moisture diffusion equation may be identical to it. So, the heat conduction and moisture diffusion equations in a saturated state may be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\kappa\frac{\mathrm{d}\check{T}(z)}{\mathrm{d}z}\right) = 0, \quad \frac{\mathrm{d}}{\mathrm{d}z}\left(\vartheta\frac{\mathrm{d}\check{C}(z)}{\mathrm{d}z}\right) = 0, \tag{12}$$

where κ and ϑ are the thermal conductivity and moisture diffusivity coefficients, respectively. The boundary conditions of temperature and moisture concentration at the upper and lower surfaces of plate are given by

$$\check{T}(z)|_{z=-h/2} = T_0, \quad \check{T}(z)|_{z=+h/2} = 3T_0,$$
$$\check{C}(z)|_{z=-h/2} = 0, \quad \check{C}(z)|_{z=+h/2} = C_0, \tag{13}$$

where T_0 is the reference initial temperature and C_0 is the reference final moisture concentration. So, the general solutions of the heat conduction and moisture concentration equations according to the above conditions are

$$\{T, C\} = \frac{1}{2\mu^2[h(y)]^2}\left\{4\left[1+\frac{z}{h(y)}\right]T_0, \left[1+\frac{2z}{h(y)}\right]C_0\right\}\sin\mu x. \tag{14}$$

So, the hygrothermal resultants M^T and M^C are given by