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Analytical solutions for crack opening displacements of eccentric cracks in thin-walled metallic plates

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ABSTRACT

In the context of the prevalence of thin-walled metallic aerospace structures, the added resistance to crack propagation offered by a built-up structure is desirable from a damage tolerance standpoint. The analysis of failure in such structures, however, is limited by the lack of crack opening solutions. This paper develops analytical models that calculate crack opening displacements (CODs) for a more general cracking scenario, i.e. non-symmetric cracks. The proposed models are based on the Westergaard stress functions. It is then found that the COD solution of one model is particularly accurate. The potential significance of the obtained solutions lies in analysing failure in built-up structures containing non-symmetric cracks. The crack opening solution is particularly useful in estimating the load transfer between cracked body and intact bridging structures in built-up structures using the principle of displacement compatibility.

1. Introduction

Built-up structures with redundant load paths offer the ability to exploit and tailor progressive failure modes within these structures. This concept is often exploited in safety critical structures within the aerospace industry where damage arresting features, such as fuselage tear straps, and inherent redundant load paths in a given structure are used to slow damage progression and enable its detection through regular inspection the Damage Tolerance philosophy [1–5]. A major key to implementing the damage tolerance philosophy is the ability to predict damage growth behaviour. The period in which damage can grow without leading to catastrophic failure defines the available inspection intervals to detect the damage. However, predicting damage growth in redundant structures requires the ability to assess the impact of the damage on the stiffness of structural elements in order to effectively determine the load redistribution using solid mechanics and the concept of displacement compatibility [6].

The classical method for analysing crack growth in many engineering materials is using linear elastic fracture mechanics (LEFM). In this approach, a linear elastic strain field is assumed in a continuum containing a crack, and this assumption is used to determine a compatible displacement field for a given load applied to the cracked body. Of primary interest for damage progression calculations is the magnitude of the singularity of the linear elastic stress field at the crack tip, or stress intensity factor (K). Analytical solutions for the displacement field and stress intensity factor exist for a wide variety of crack configurations [7]. For crack configurations for which direct analytical solutions are not available, geometrical correction factors to modify the analytical solution for a crack in an infinite plate are often generated through experimental and/or numerical methods. These correction factors for stress intensity factor, known as β factors, are widely available in the literature [7–12]. However, corrections for the displacement field are often not available.

The accuracy of β factors developed by Isida is better than 1% [7]. The β factors were developed by expressing the Airy's stress functions in terms of complex potential functions and solving these potential functions [12]. The β factors are expressed as functions of the coefficients which are tabulated in [12]. However, the accurate β factors cannot be directly applied for an eccentrically cracked panel containing stiffening elements. The interplay between the eccentrically cracked panel and the redundant load paths must be taken into consideration in calculating the stress singularity in front of the crack tips.

The objective of this paper is to develop means of correcting the Mode I crack opening displacement (COD) of non-symmetric cracks in thin-walled metallic panels using linear elastic fracture mechanics. The inherent assumptions of LEFM have to be respected when applying the developed approach. The proposed approach is envisioned to be applicable to thick panels where the state of plane strain is valid.

The purpose of developing such correcting methods is to assist the analysis of failure in eccentrically cracked panels with stiffening elements. This paper proposes 4 analytical models based on the Westergaard stress function [13]. The Westergaard stress function is

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Nomenclature

a	Half Crack length (mm)
a_1	Distance between maximum crack opening location and crack tip 1 (mm)
a_2	Distance between maximum crack opening location and crack tip 2 (mm)
b	Delamination (mm)
d	Distance between crack centre and panel centre (mm)
d_1	Distance between the location of P_1 and panel centre (mm)
d_2	Distance between the location of P_2 and panel centre (mm)
e	Eccentricity
E	Young's modulus (MPa)
E_f	Young's modulus of fibre layer (MPa)
K	Stress intensity factor ($MPa\sqrt{mm}$)
K_{br}	Stress intensity factor for crack tip 2 ($MPa\sqrt{mm}$)
K_{total}	Total stress intensity factor of the metal layer in Glare panels ($MPa\sqrt{mm}$)
L	Distance from crack tip to the free edge ahead (mm)
P	Integral of stress distribution in front of crack tip (N)
P_{app}	Total applied load (N)
S_{br}	Bridging stress distribution (MPa)
S_f	Stress in the fibre layer due to applied load (MPa)
S_m	Stress in the metal layer due to applied load (MPa)
t	Thickness (mm)
w	Distance between crack datum and free edge (mm)
W	Panel width (mm)
x	Horizontal location in a xy-coordinate system (mm)
x_c	Centroid of the stress distribution in front of the crack tip (mm)
y	Vertical location in a xy-coordinate system (mm)
σ	Applied tensile stress (MPa)
σ_{yy}	Stress distribution in front of crack tip in loading direction (MPa)
τ_{xy}	shear stress (MPa)
λ	Normalized crack length
β	Correction factor for stress intensity factor
β_{Isida}	Correction factor for stress intensity factor derived by Isida
v	Crack opening displacement (mm)
v_{ff}	Crack opening displacement due to far-field stress (mm)
v_{br}	Crack opening displacement due to bridging stress (mm)
δ_f	Fibre elongation (mm)

Subscripts

1	referring to crack tip 1
2	referring to crack tip 2

simplified to provide solutions for the crack opening displacement and stress-strain field ahead of a non-symmetric crack. The stress intensity factor solutions arising from the simplified Westergaard stress function are compared to the results of Isida to show the validity of the proposed models. A validated Finite Element Modelling (FEM) technique is applied to obtain the COD and strain field in front of non-symmetric cracks, the simulation results are used to screen the 4 models in Section 4. In Section 5, the opening displacement solution will be employed in a simplified case study to evaluate the impact of the load transfer mechanism on the stress intensity factors.

2. The westergaard function method

The Westergaard function method is a very convenient methodology to characterize the entire stress and strain fields for a cracked body. The Westergaard functions can also be simplified to their near-tip solutions, i.e. stress intensity factor (SIF) solutions which provide the stress and strain distributions at the crack tip vicinities [7,14]. In some instances, however, it is desirable to know the entire stress-strain field ahead of the crack tip. Load redistribution due to stiffness variation (either geometric or material stiffness) can be resolved from such a stress-strain field, such as the analysis of crack growth behaviour in a stiffened panel conducted by Rans [6].

The closed-form Westergaard solutions are strictly applicable to infinite plate crack problems except for Mode III crack problems [7]; nevertheless they can be modified to provide meaningful solutions for a finite panel with a crack [15,16,6]. Barsoum et al. [15,16] predict the stress intensity factor for cracks in finite width functionally graded material containing layers with different stiffness using this method. With the same assumption, Rans [6] predicts the crack growth in stiffened metallic panels.

Consider a central crack embedded in a finite panel under uniform tensile loading, a Mode I crack problem, the stress ahead of the crack tip along crack plane can be assumed to follow the Westergaard stress distribution [7,11]:

$$\sigma_{yy} = \frac{\sigma\cdot\beta}{\sqrt{1-(a/x)^2}}. \tag{1}$$

where σ is applied stress, β is a correction factor, a is the crack length and x the distance from the crack centre in the crack plane.

The introduction of the correction factor β is to account for the influence of the finite width boundary condition. The correction factor β is a function of crack length and panel width. This variable can be calculated using the load equilibrium between the crack section and far-field. For a finite panel of width W, the load equilibrium can be expressed as:

$$\sigma\cdot W = 2\int_a^{W/2} \frac{\sigma\cdot\beta}{\sqrt{1-(a/x)^2}}dx \tag{2}$$

Solving the integral for a uniform stress state and rearranging for β:

$$\beta = \frac{1}{\sqrt{1-(2a/W)^2}} \tag{3}$$

The corresponding crack opening displacement for plane stress state can be given by:

$$2v = \frac{4\beta\sigma}{E}\sqrt{a^2-x^2} \tag{4}$$

and stress intensity factor is expressed as following:

$$K = \beta\sigma\sqrt{\pi a} \tag{5}$$

The correction factor β is plotted in Fig. 1 against the ratio between the crack length and specimen width. In order to show the validity of the solution from the simplified Westergaard stress function, the obtained correction factor for SIF is compared to that derived by Isida. see Fig. 1. Good correlation can be observed.

It is worth noting that a finite panel with a central crack under Mode I loading possesses a symmetric axis passing through the crack centre in loading direction. The Westergaard stress distributions in front of two crack tips, crack opening displacement configuration are symmetric with respect to this symmetric axis (or crack centre). Especially it can be seen in Eqs. (1), (4), (5) that these variables are functions of the crack length a which is measured from the crack centre, also the location of maximum crack opening, to the crack tip.

The presence of an eccentric crack in a finite panel under Mode I loading eliminates the symmetric condition possessed by a centrally cracked panel. As a result, the COD configuration and the stress distributions (stress-strain fields) ahead of the non-symmetric crack tips