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Symmetric and antisymmetric lateral–torsional buckling of prestressed steel I-beams

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ABSTRACT

Until now, there are no analytical solutions published of the lateral-torsional buckling (LTB) of the prestressed steel I-beams. In this paper, the symmetric and antisymmetric elastic LTB buckling performance of prestressed steel I-beams, with rectilinear tendons, under equal end moments has been examined. Firstly, a simplified mechanical model is put forward. Secondly, based upon the Euler-Beam model & the Kirchhoff-Plate model, the strain energy equation as well as the potential energy of the prestressed steel I-beam are derived in detail. Then the total potential equation and the differential equilibrium equations along with the boundary conditions are obtained. As a result, the analytical solutions of the symmetric and antisymmetric elastic LTB of prestressed I-beams are obtained, for the first time, and the correctness of the analytical solution of the critical moment of the symmetric buckling of the prestressed I-beam is verified by those simulated by using ANSYS. Finally, parameter analysis is carried out. It is found that: (1) the critical moment of the prestressed steel I-beam, with rectilinear tendons, under equal end moments can be determined by that of the symmetric buckling; (2) the critical prestressing force can be determined by that of the symmetric buckling or antisymmetric buckling depending on the relationship between the dimensionless eccentricity and its threshold value.

1. Introduction

Nowadays, prestressing techniques have been widely considered in the construction of long span structures [1].

The basic principles of prestressing is that of the introduction of internal stress of such magnitude and distribution that stresses resulting from given external loadings are counteracted to a desired degree. Similar principle of analysis may be applied to prestressed steel beams. In contrast to prestressed concrete beams, whose cross section cannot take tensile stress, the prestressed steel beam does not require special stress distribution. In addition, the losses due to friction between the tendons and holes/collars are smaller than the prestressed concrete beam. Therefore, in the case of prestressed steel beam the possible savings in structural steel weight has been estimated to be in the range of approximate 10–30% [2], as compared with conventional structural steel design. In addition, prestressing of a steel beam also considerably reduces its deflections induced by the service loads, accordingly it enhances the service behavior of the steel beam from the serviceability limit state point of view. The structural efficiency, economy and design flexibility of prestressed steel beams have been demonstrated in their extensive use in engineering projects.

Prestressing of steel structures has been the subject of a number of investigations. Early work by Magnel [3] experimentally demonstrated the improved economy that can be achieved by prestressing truss girders. In China, Prof. Zhong [4,5] and Prof. Lu [6] et al. have explored a series theoretical and experimental investigations on the behavior and design of prestressed steel beams, columns, trusses and space structures. However, there are few studies on the buckling behavior of prestressed steel structures so far. However, recently, the issue of buckling behavior of the prestressed stayed columns has aroused great interest due to the need for engineering applications, and there are many theories, numerical simulation and experimental results have been published [7–14], and a thorough literature review can be found in Serra et al. [11].

It is well known that prestressing to a steel beam changes the loading condition from a beam to a beam -column, changing drastically its structural behavior. Therefore the buckling behavior of the prestressed steel beam during or after the prestressing operation is more complicated than the conventional steel beam without external tendons, due to the indeterminacy provided by tendons.

As per the available literature, the bending behavior of prestressed concrete beams and prestressed steel-concrete composite beams had been thoroughly investigated and studied; however that is not the case for prestressed steel beams [15]. Moreover, it is found that few has investigated the buckling behavior of the prestressed steel beams, and only the buckling behavior during the prestressing operation is

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investigated.

Prof. Zhong [5] believed that the buckling could occur during tension, which was a potential hazard and should be checked in the two stage design method of the prestressed steel beam. He has suggested an overall stability checking method for the prestressed steel beam using concept of “Vlasov's Circle of Stability” under eccentric prestressing force. Similar methods were also used and republished by Prof. Gupta [16]. Unlike previous studies, Bradford [17] proposed design charts for the elastic buckling load of prestressed steel plate girders induced by stressing an eccentric tendon, and uses this to obtain design buckling strength in accordance with the LRFD Specification. Belletti and Gasperi [18] carried out nonlinear finite-element analyses to investigate the behavior up to failure of prestressed steel beams. The numerical results of the elastic buckling load induced by stressing an eccentric tendon is briefly reported.

Abdelnabi [19]carried out nonlinear finite-element analyses to investigate the buckling strength of externally prestressed steel plate girders. A design method was proposed based upon the concept of beam-column criteria of steel I-beams. Three loading types were considered in his investigations, i.e. uniform bending moment load, line load and single concentrated load at mid-span.

Unfortunately, the proposed elastic buckling formula of the steel beam during the prestressing operation does not consider the influence of intermediate stiffeners (deviators), which may be used to give the tendons a specific configuration. Moreover, no theoretical solution was put forward for the elastic buckling of the prestressed steel beam after the prestressing operation. Obviously, these two problems are the last hurdle for the promotion and application of the prestressed steel beams. There is no doubt that solving these problems theoretically and experimentally will provide the scientific guidance for the engineering design of prestressed steel beams and will facilitate the application of such prestressed structures.

The main subject of the present paper is the formulation of the analytical solution for the elastic lateral-torsional buckling (LTB) of the prestressed steel beam with the doubly symmetric sections during and after the prestressing operation, in which the resulting LTB energy equation is formulated based on the Euler-beam model & Kirchhoff-plate model, and with symmetric and antisymmetric buckling modes of the externally prestressed I-beams being taken into account to understand the buckling characteristics better during and after the prestressing operation.

In this paper, the simplified mechanical model of the prestressed steel I-beam with doubly symmetric section and rectilinear prestressed tendons is depicted in Section 2. The energy equation of the prestressed steel I-beam is derived via an Euler-Beam model and a Kirchhoff-Plate model in Section 3, while the total potential equation and differential equilibrium equations along with the boundary conditions are presented in Section 4. Analytical solutions of the symmetric and antisymmetric elastic LTB of prestressed I-beams are obtained and discussed in Section 5, while comparisons between the FEM numerical simulation and analytical results are presented in Section 6, followed by parameter analysis and design proposals.

2. Simplified mechanical model

2.1. Calculating diagram

A simply supported, prestressed steel I-beam with a doubly symmetric section, as shown in Fig. 1, is selected as the research object. This beam is subjected to equal and opposite end moments, i.e. the prestressed steel I-beam is in the state of pure bending in the vertical plane of the beam. At this time, the stress distribution on the beam cross section is constant along the beam length.

Only elastic buckling behavior is taken into account in this paper. The elastic modulus of elasticity and the shear modulus of elasticity of structural steel are taken as E and G, respectively.

The tendons of high-tensile steel used for prestressing usually the take one of the following form: cable, wire ropes, strands, and bars. Configuration of the tendons may be one of the following: rectilinear, curvilinear and polygonal. Perhaps the easiest way to prestress a steel I-beam is to use straight high-strength cables [17], which are anchored at the ends of the beam, and stressed in a manner analogous to that for prestressing concrete beams. Therefore, this case, as shown in Fig. 1, will be considered in this investigation.

It is assumed that when the prestressing process is complete the tendons and the steel beam are locked in position at both ends such that there is no relative movement between the two components.

To increase the buckling strength of the steel I-beam during the prestressing operation, additional intermediate stiffeners (called "deviators" for curvilinear and polygonal configuration) may be used. The tendons should close contact with the holes/collars of the intermediate stiffeners deviators) to give the tendons a specific configuration. In practice, the gap (usually 1–2 mm) between the holes/collars and the tendons should be as small as possible, allowing the tendons in the holes can be free to stretch in the longitudinal direction, but also to ensure that the tendons and the bar in the transverse direction to work together. Hence it is further assumed that when the prestressed steel beam is buckled the transverse displacement of the centroid of the tendons contact with the stiffener is the same as that of the holes/collars. For the sake of simplicity, this paper only considers the case where there is an intermediate stiffener in the middle of the span of the prestressed steel beam, as shown in Fig. 1.

2.2. Deformation model: Euler-beam model and Kirchhoff-plate model

It is noteworthy that almost all published literature [20–30] uses the section warping function proposed by Vlasov to describe the LTB deformation of a steel beam. However, the section warping function has two drawbacks, the first is that the St-Venant's torsion can not be deduced from this warping function naturally; the second is that this function can not be utilized directly to the inelastic LTB problem in which the influence of different material modulus should be considered. In order to overcome these problems, the transformed section method has been proposed in Chen [28] and has also been described in Trahair's book [24] in the theoretical framework of Vlasov. It has been found and proved by the author [32] that in general the warping rigidity will be significantly underestimated by the transformed section method, and hence the buckling load given by the transformed section method maybe only 1/5 of that predicted by the Plate-beam theory. In other words, the transformed section method does not appear to lead to the correct prediction of the elastioplastic lateral buckling in some cases (more related analytical results will be published elswhere). This may be why there are so large differences in world's steel beam design specifications. Therefore, the section warping function should be abandoned for the inelastic LTB problem of steel beam or the elastic LTB problem of steel-concrete composite thin-walled structures.

Generally, steel beams (e.g. I-beams or T-beams) are composed of three or two flat plates, so how to use the classical engineering mechanics to describe the in-plane deformation and the out-of-plane deformation of each flat plate (either a web or flange plate) is the key to a more rational solution to the LTB problem. It is just based on such a basic understanding that the author created and put forward the Plate-Beam Theory recently [31–37]. The following two deformation hypotheses (Fig. 2) are used in this new theory.

(1) Hypothesis of rigid cross-section. That is, the contour of the cross-section is undeformable, i.e. the cross-section is rigid after the occurrence of the beam lateral buckling.

This is the well-known peripheral rigidity hypothesis of Vlasov, which means that the local and distortional buckling are excluded in the LTB model;

(2) Hypothesis of neglecting shear deformation. That is, the