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Suppressing vibrational response of functionally graded truncated conical shells by active control and design optimization

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A B S T R A C T

A shear deformation shell theory including thickness stretching effects is used to formulate the minimization problem of the vibrational response of functionally graded truncated conical shells in different cases of boundary conditions. Mechanical control energy is introduced into the formulation as a functional containing a closed-loop control force. The optimization objective is taken as the sum of the control energy and the total energy of the shell. Based on Lyapunov–Bellman theory, optimum values for the control forces and deflections are obtained for shells with simply supported or clamped edges. A design procedure is applied to complete the minimization process for the control objective using material and geometric parameters. Numerical and graphical results are presented to show the importance of the inclusion of the thickness stretching effects into the formulation. An assessment for the current design and control approach in minimizing the optimization objective is performed.

1. Introduction

Conical and cylindrical composite shells are extensively used in many industrial and engineering structures such as aerospace, aeronautical, naval, and civil structures. For large space structures, it is required light materials with very high flexibility and low natural damping. One of the dominate problems facing the designers of such structures is suppressing excessive vibrations [1,2]. Functionally graded materials (FGMs) are a kind of composites in which the macroscopic mechanical properties vary continuously and smoothly as the dimension varies [3–6]. These advanced composites possess numerous characteristics such as reduction of stress concentration, high toughness, improved thermal properties, etc. which make them appropriate for tailoring a composite material to optimize a desired characteristic [7,8].

The active and passive control for suppressing the excessive vibrations in large space structures is considered one of the effective means for damping the vibrational response of these structures [9–11]. Several mathematical models are available in the literature and dealing with the design and control optimization for composite beams, plates and shells [12–15]. Many multiobjective optimization approaches for these components are presented with constraints imposed on relevant quantities, see e.g. [16–19], but, relatively little studies have been devoted to the optimization problems of truncated conical shells, particularly, conical shells made of FGMs. Moreover, many studies have indicated that neglecting shear and normal strains may lead to high errors in predicting the optimal ranges for the design and control parameters [20–24]. However, the studies dealing with the optimization problems of composite shells and including these effects are few.

The present study is concerned with an optimal control problem of minimizing the vibrational (dynamic) response of composite FG truncated shells with the lowest expenditure for the control energy. The problem is formulated based on a shell theory accounting for the effects of shear deformation and normal strains, and a shear correction factor is used. The total energy of the shell is considered as criteria for the shell dynamic response. The cost objective of the control problem is assumed to be a functional including the total elastic energy and control energy due to a closed loop control force acting on the outer surface of the shell. Based on Lyapunov–Bellman theory [25], optimum values for the control force and deflections are obtained for shells with simply supported or clamped edges. A design procedure is used to lower the level of the minimized energy of the shell using material and geometric parameters as design variables. Numerical and graphical results are presented to assess the inclusion of the thickness normal strain effect into the shell optimization problems. The effectiveness of the present control and design approach in minimizing the vibrational energy of the conical shells is examined.

2. Governing and constitutive equations

A functionally graded truncated circular conical shell is composed of two orthotropic materials such that the shell macroscopic mechanical properties are orthotropic, and vary continuously across the shell

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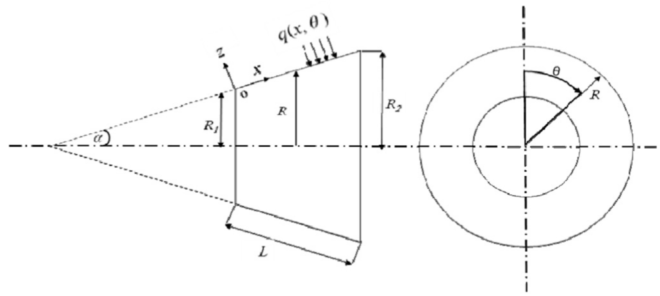

Fig. 1. The geometry and coordinates of the truncated conical shell.

thickness. The shell is of a uniform thickness h, a semivertex angle α, a meridional length L along the generator of the cone, and R_1 and R_2 are the radii of the cone at the small and large basses, respectively. Introduce a curvilinear coordinate system $x\theta z$ with origin on the shell middle surface at the small base of the shell as shown in Fig. 1. The coordinates x and z are in the directions of the generator and thickness of the conical shell, respectively, and the coordinate θ is an angle in the circumferential direction. Then, the radius R of the cone in the radial direction at any point is:

$$R = R_1 + x\sin(\alpha). \tag{1}$$

The present study accounts for a shell theory involving shear deformation and normal strain effects, and the displacement field may be taken as:

$$\begin{aligned} u_x &= u + z\psi,\\ u_\theta &= v + \phi z,\\ u_z &= w_0 + \beta(w_1 z + w_2 z^2), \end{aligned} \tag{2}$$

where $(u_x,\ u_\theta,\ u_z)$ are displacements in the directions of the coordinates x, θ and z, respectively, (u, v, w_0) are displacements of a point on the middle surface of the shell, and (w_1, w_2) are unknown functions with no geometric meaning. The inclination angles $(\psi,\ \phi)$ determine slops of the normal to the shell middle surface in xz and θz surfaces due to bending only. β is a parameter taking the values 0 or 1 for studying the influence of the inclusion of normal strain into the shell control problems.

The strains ε_{ij} corresponding to the displacements (2) may be obtained in the form:

$$\begin{aligned} &\varepsilon_x = u_{x,x}, \quad \varepsilon_z = u_{z,z}, \quad \gamma_{xz} = u_{x,z} + u_{z,x},\\ &\varepsilon_\theta = [(\sin\alpha + \cos\alpha)\, u_x + u_{x,\theta}]/R,\\ &\gamma_{x\theta} = (u_{x,\theta} + u_{\theta,x} - u_x \sin\alpha)/R,\\ &\gamma_{\theta z} = (u_{z,\theta} + u_{\theta,z} - u_\theta \cos\alpha)/R, \end{aligned} \tag{3}$$

where comma (,) means a partial differentiation with respect to the followed coordinate.

The constitutive relations for an orthotropic shell are

$$\begin{bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_z\\ \tau_{x\theta}\end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{16}\\ Q_{12} & Q_{22} & Q_{23} & Q_{26}\\ Q_{13} & Q_{23} & Q_{33} & Q_{36}\\ Q_{16} & Q_{26} & Q_{36} & Q_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{x\theta}\end{bmatrix}, \quad \begin{bmatrix}\tau_{xz}\\ \tau_{\theta z}\end{bmatrix} = \begin{bmatrix} Q_{44} & Q_{45}\\ Q_{45} & Q_{55}\end{bmatrix} \begin{bmatrix}\gamma_{xz}\\ \gamma_{\theta z}\end{bmatrix}. \tag{4}$$

Here, $(\sigma_x, \sigma_\theta, \sigma_{x\theta})$ and $(\tau_z, \tau_{xz}, \tau_{\theta z})$ are the in-surface and transverse shear stresses, $(\varepsilon_x, \varepsilon_\theta, \varepsilon_z, \gamma_{x\theta}, \gamma_{xz}, \gamma_{\theta z})$ are the strain components, and Q_{ij} are the stiffness coefficients which are functions of the thickness coordinate z.

Let the outer surface of the conical shell be subjected to a distributed load $q(x,\ \theta,\ t)$ acting as a control force. Using the dynamic version of the principle of virtual displacements, the governing equations of the shell may be obtained as follows:

$$\begin{aligned} &N_{xx,x} + N_{x\theta,\theta}/R + (N_{xx} - N_{\theta\theta})\sin\alpha/R\\ &\quad = I_0\ddot{u} + I_1\ddot{\psi},\\ &N_{x\theta,x} + (N_{\theta\theta,\theta} + 2N_{x\theta}\sin\alpha + Q_{\theta\theta}\cos\alpha)/R\\ &\quad = I_0\ddot{v} + I_1\ddot{\phi},\\ &Q_{xx,x} + (Q_{\theta\theta,\theta} + Q_{xx}\sin\alpha - N_{\theta\theta}\cos\alpha)/R + q\\ &\quad = I_0\ddot{w}_0 + \beta(I_1\ddot{w}_1 + I_2\ddot{w}_2),\\ &M_{xx,x} + M_{x\theta,\theta}/R + (M_{xx} - M_{\theta\theta})\sin\alpha/R - Q_{xx}\\ &\quad = I_1\ddot{u} + I_2\ddot{\psi},\\ &M_{x\theta,x} + M_{\theta\theta,\theta}/R + 2M_{x\theta}\sin\alpha/R - Q_{\theta\theta}\\ &\quad = I_1\ddot{v} + I_2\ddot{\phi},\\ &K_{xx,x} + (K_{\theta\theta,\theta} + K_{xx}\sin\alpha - M_{\theta\theta}\cos\alpha)/R\\ &\quad + hq/2 - N_{zz} = I_1\ddot{w}_0 + \beta(I_2\ddot{w}_1 + I_3\ddot{w}_2),\\ &F_{xx,x} + (F_{\theta\theta,\theta} + F_{xx}\sin\alpha - R_{\theta\theta}\cos\alpha)/R\\ &\quad + h^2q/4 - 2M_{zz} = I_2\ddot{w}_0 + \beta(I_3\ddot{w}_1 + I_4\ddot{w}_2), \end{aligned} \tag{5}$$

where the force and moment resultants $N_{xx}, N_{x\theta}, N_{\theta\theta}, \dots, F_{\theta\theta}$ are defined as:

$$\begin{aligned} &\begin{bmatrix} N_{xx} & M_{xx} & 0\\ N_{\theta\theta} & M_{\theta\theta} & R_{\theta\theta}\\ N_{x\theta} & M_{x\theta} & 0\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\end{bmatrix} [1 \quad z \quad z^2]\, dz,\\ &\begin{bmatrix} Q_{xx} & K_{xx} & F_{xx}\\ Q_{\theta\theta} & K_{\theta\theta} & F_{\theta\theta}\end{bmatrix} = \varepsilon\int_{-h/2}^{h/2} \begin{bmatrix}\tau_{xz}\\ \tau_{\theta z}\end{bmatrix} [1 \quad z \quad z^2]\, dz, \end{aligned} \tag{6}$$

where ε is a shear correction factor. In the present study, three types of edges conditions are considered, when the two bases of the shell are simply supported (*SS*), clamped (*CC*) and mixed of them (*CS*), and these boundary conditions are:

$$\begin{aligned} &S:\ u_0 = w_0 = w_1 = w_2 = \phi = N_{x\theta} = M_{x\theta} = 0,\\ &C:\ u_0 = w_0 = w_1 = w_2 = \psi = \phi = N_{x\theta} = 0. \end{aligned} \tag{7}$$

The governing equations of the cylindrical and annular shells may be obtained, respectively, by setting $\alpha = 0$ and $\alpha = \pi/2$ in Eqs. (3)–(6).

3. The control and design objective

The control procedure is to minimize the vibrational (dynamic) response of a FG conical shell due to initial disturbances given by:

$$w(x, \theta, 0) = \overline{A}(x, \theta), \quad \dot{w}(x, \theta, 0) = \overline{B}(x, \theta), \tag{8}$$

in an interval of time $0 \le t \le \tau \le \infty$ with the lowest expenditure for the control energy due to a closed loop control load $q(x,\ \theta,\ t)$ distributed on the outer surface of the conical shell. The shell total energy may be considered as a criteria for the vibrational response, therefore, the cost function J for the control problem may be written as:

$$\begin{aligned} &J = \mu_1 J_1 + \mu_2 J_2 + \mu_3 J_3,\\ &J_1 = \frac{1}{2}\int_0^\tau \int_0^L \int_0^{2\pi} \int_{-h/2}^{h/2} (\varepsilon_z\ \sigma_z + \varepsilon_\theta\ \sigma_\theta + \varepsilon_x\ \sigma_x + \gamma_{x\theta}\ \tau_{x\theta}\\ &\quad + \gamma_{\theta z}\tau_{\theta z} + \gamma_{xz}\tau_{xz})\ R\ dz\ d\theta\, dx\ dt,\\ &J_2 = \frac{1}{2}\int_0^\tau \int_0^L \int_0^{2\pi} \int_{-h/2}^{h/2} \rho\ (\dot{u}_x^2 + \dot{u}_\theta^2 + \dot{u}_z^2)\ R\ dz d\theta dx dt,\\ &J_3 = \int_0^\tau \int_0^L \int_0^{2\pi} q^2(x_1,\ x_2,\ t)\ R\ d\theta\ dx\ dt, \end{aligned} \tag{9}$$

where $(\mu_1,\ \mu_2,\ \mu_3)$ are positive weighting coefficients, J_1 and J_2 represent the deformation and kinetic energies of the conical shell, respectively. The functional J_3 represents a control energy including the control function q, $q \in D^2$, D^2 is the set of all solutions which must be bounded, quadratic and integerable on the domain $\{0 \le x \le L,\ 0 \le \theta \le 2\pi,\ 0 \le t \le \tau \le \infty\}$. In addition, the cost function J is quadratic and positive definite. Therefore, it is differentiable on the domain D^2. Then, the solution of the control problem may be reduced to find the optimum control force $q(x,\ \theta,\ t)$ from the initial boundary-value problem (5)–(8) with the condition of minimizing the cost functional (9). Furthermore, a design procedure is carried out to lower the level of the minimized total energy of the shell using material and