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Experimental verification on simplified estimation method for envelope curve of wrinkled membrane surface distortions

Takashi Iwasa

Department of Mechanical and Aerospace Engineering, Tottori University, 4-101, Koyama-cho Minami, Tottori 680-8551, Japan

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A B S T R A C T

The effectiveness of a simplified method for estimating an envelope curve of wrinkled-membrane-surface distortion, which was recently proposed by the author, was experimentally assessed. The equation is formulated using three physical quantities regarding wrinkles: the length of the wrinkle line, the major principal strain in wrinkled regions, and the in-plane shrinkage strain appearing in the orthogonal direction of the wrinkle line. Since these three physical quantities are attributed to two-dimensional problems, the formula makes it possible to simply estimate wrinkle amplitude without a cumbersome bifurcation analysis. Applying a wrinkle strain in tension-field theory instead of the in-plane shrinkage strain in the formula, a simplified method to estimate an envelope curve of a wrinkled membrane with a low computational cost was developed using the wrinkling analysis with tension-field theory. Wrinkling phenomena appearing on two membrane models subjected to an in-plane shear and a corner-tension load were experimentally measured by photogrammetry using the direct linear-transformation method and a laser-displacement sensor. The experiment model was then subjected to a finite element analysis using tension-field theory, and an envelope curve of the wrinkled membrane was estimated using the method. The estimated envelope curves appropriately captured the actual wrinkle amplitude appearing on the membrane surface regardless of the formation process of wrinkles. From the result, the validity of the proposed estimation method was confirmed. This paper offers an effective method to predict the magnitude of the wrinkled-membrane-surface distortion with a low computational cost, and will assist the development of future gossamer space structures incorporating membranes.

1. Introduction

A membrane is a lightweight, flexible structure that can be packaged easily into a small stowed volume and deployed into a shape with a large surface area. Because of their distinctive mechanical properties, membranes have been used as major structural components of gossamer space structures such as solar sails [1], inflatable antenna reflectors [2] and sunshields [3]. However, membranes have negligible bending stiffness and cannot resist any compressive loading. Once a membrane experiences a compressive stress due to an external disturbance, it immediately buckles, giving rise to wrinkles on its surface [4]. This wrinkling phenomenon has a negative effect upon the operation of gossamer space structures [5,6]. Hence, it is important to be able to predict it accurately in order to design and build future structures reliably.

Currently, the prediction of wrinkled membrane behavior in gossamer space structures depends on computational approaches. This is because these structures deform easily under gravity, and so membrane behavior such as wrinkled and slacked regions are difficult to verify from ground tests. Computational approaches to wrinkled membrane behavior are generally classified into two categories: post-buckling analysis using shell theory [7–14], or tension-field theory [15–25]. Of these, post-buckling analysis using shell theory reveals the wrinkle geometry (e.g., wavelength and amplitude of wrinkles) and is suitable for predicting wrinkled membrane behavior in quantitative detail. However, shell-theory analysis is computationally expensive for obtaining the final solution. It is also difficult to be applied as a practical tool for predicting wrinkled membrane behavior of a gossamer space structure in its design stage. In contrast, the analysis using tension-field theory predicts the overall wrinkled membrane behavior with low computational cost, but can reveal only the stress and strain fields on the wrinkled membrane. Since this form of analysis cannot calculate the wrinkle geometry, it suffers from being unable to establish the magnitude of the surface distortion. Accordingly, there has long been a demand for a practical and computationally inexpensive method of estimating the magnitude of the surface distortion of a wrinkled membrane.

To solve this problem, a simplified equation to calculate an

E-mail address: iwasa@mech.tottori-u.ac.jp.

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envelope curve of the wrinkled membrane was proposed [26]. The equation was formulated based on Wong and Pellegrino's analytical model [27] and can be used to calculate the maximum wrinkle amplitude based on three physical quantities: the wrinkle length, the major principal strain in a wrinkled region, and the in-plane shrinkage strain orthogonal to a wrinkle. Since these physical quantities are associated with two-dimensional (2D) problems, the maximum wrinkle amplitude can be obtained simply from the formula without a cumbersome bifurcation analysis. Accordingly, the proposed equation can easily estimate the magnitude of wrinkled-membrane-surface distortion, although it cannot reveal the exact wrinkle features. By incorporating the formula and the wrinkling analysis using tension-field theory, a simplified method for estimating the envelope curve of a wrinkled membrane was developed, and its validity was confirmed by comparing the estimation results with the simulation results of shell-theory analysis [26]. However, to demonstrate the practicality of the estimation method for actual membrane structures, it is necessary to compare the estimation results with actual wrinkled-membrane behavior. Thus, in this study, by comparing the estimated envelope curve with the experimentally observed wrinkled membrane, the practicality of the estimation method for actual membrane behavior is assessed. In this assessment, two membrane models were treated. One is a square membrane subjected to a shear load, and the other is a square membrane subjected to a corner-tension load. The experimental data for the latter model were taken from a previous study [28].

The reminder of this paper is structured as follows. In Section 2, we derive the formula for wrinkle amplitude and explain the method for estimating the envelope curve of a wrinkled membrane surface. In Section 3, we outline the experiment, including the photogrammetric-measurement system and the laser-displacement-sensor measurement system [28] and describe in detail the experiment model treated in this study. In Section 4, we outline the wrinkling analysis based on tension-field theory, describe the finite element analysis with a special membrane element based on tension-field theory, and state the analysis procedure for the experimental model. In Section 5, we compare the analytical results with the experimental ones and discuss the validity of the method for estimating the envelope curve of a wrinkled membrane. Finally, in Section 6, we summarize the major findings of this study.

2. Estimating the envelope curve of a wrinkled membrane surface

2.1. Three assumptions

To formulate an equation for the amplitude of a membrane wrinkle, we make the following three assumptions based on Wong and Pellegrino's [27] analytical model. Fig. 1 illustrates the single-wrinkle model used in the assumptions.

1) The occurrence of a single wrinkle is modeled as the buckling of a simply supported finite plate whose side lengths are the length l_ξ of the wrinkle line and the half-wavelength l_η of the wrinkle.
2) The minor principal stress in the wrinkled region is always equal to the buckling stress σ_{cr} of the plate model.
3) Wrinkling behavior in the direction of minor principal stresses obeys inextensional theory.

In Wong and Pellegrino's analytical model, the appearance of a single wrinkle is modeled as the buckling of an infinite plate. However, in our study, a simply supported finite plate model is applied so that the formulated equation can treat realistic wrinkles with finite length for various membrane structures.

2.2. Formula for the envelope curves of wrinkles

The formulated equation is intended for estimating the amplitude of wrinkles appearing on an isotropic membrane. Although some papers treating the wrinkle amplitude use Föppl-von Kármán plate theory [4,29], this paper applies an equilibrium equation not to include bending stiffness term to calculate the wrinkle amplitude. In this equation, since the in-plane membrane strains are completely decoupled from the out-of-plane wrinkling deformation, the proposed method connects the in-plane strain to the out-of-plane wrinkling deformation by assuming that the minor principal stress in the wrinkled region is equal to the buckling stress of the plate model. According to this assumption, the formulated equation makes it possible to simply estimate an approximate wrinkle amplitude from tension-field solutions without a cumbersome bifurcation analysis. In addition, by explicitly incorporating the tension-field solutions to the formula stated below, the derived equation can be applied to various types of membrane structures.

Focusing on a finite region of the wrinkled membrane surface, as shown in Fig. 1, an equilibrium equation in the out-of-plane direction is given by

$$\sigma_\xi \kappa_\xi + \sigma_\eta \kappa_\eta = 0 \tag{1}$$

where κ_ξ and κ_η are expressed as

$$\kappa_\xi = -\frac{\partial^2 w}{d\xi^2},\ \kappa_\eta = -\frac{\partial^2 w}{d\eta^2}. \tag{2}$$

From the first assumption above, the compressive stress causing a wrinkle is given by the critical buckling stress of the simply supported finite plate, namely,

$$\sigma_{cr} = -\frac{\pi^2 D}{t {l_\xi}^2}\left(\frac{l_\xi}{l_\eta} + \frac{l_\eta}{l_\xi}\right)^2 \tag{3}$$

Here, t is a membrane thickness and D is given by

$$D = \frac{Et^3}{12(1-\upsilon^2)} \tag{4}$$

where E is a young's modulus and υ is a Poisson's ratio. The surface shape of the single wrinkle is represented by the first buckling mode of the simply supported finite plate. Thus, an out-of-plane displacement of the single wrinkle w is given by

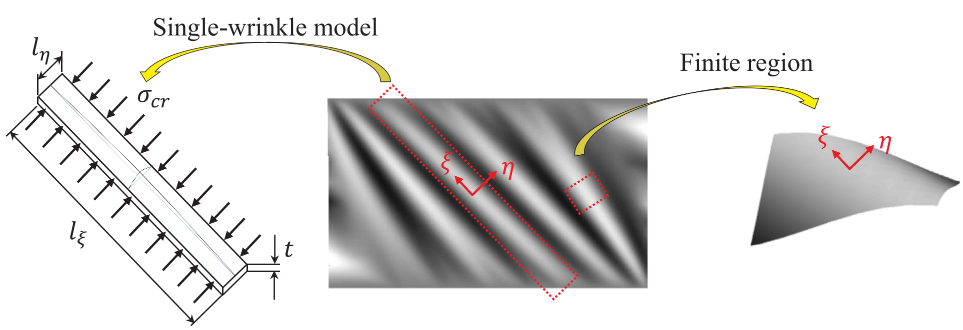

Fig. 1. Outline of single-wrinkle model. Appearance of a single wrinkle is modeled as the buckling of a simply supported finite plate subjected to compressive stresses. The cross-sectional shape of a single wrinkle in the direction of minor principal stress is assumed to be a sine wave; ξ and η are the local coordinate axes in the directions of major and minor principal stresses, respectively. The wrinkled membrane surface feature was generated by finite element analysis using shell theory.