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Vibrational behavior of variable section functionally graded microbeams carrying microparticles in thermal environment

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ABSTRACT

Based on the modified strain gradient theory (MSGT) in conjunction with the Timoshenko beam theory, the vibrational behavior under thermal environment of variable section functionally graded (FG) microbeams carrying microparticles is studied. The eigenfrequency equations are derived using the Chebyshev–Ritz method. The effects of temperature dependence of material properties and the initial thermal stresses are considered. After validating the method, the influences of the width taper ratio, the position and values of the microparticle masses, temperature rise, length scale parameters and material gradient index on the free vibration behavior of microbeams are studied. In addition, comparisons between the results of the MSGT, the modified couple stress theory (MCST), and the classical theory (CT) are performed. It is found that in addition to the microparticle masses and locations, their influences on the natural frequencies depend on the vibration mode number and the edge restraints of microbeams. Also, it is shown that the temperature rise, length scale parameters and material gradient index have considerable effects on the natural frequencies of microbeams. Moreover, the results demonstrate that the MSGT and the CT provide the largest and the smallest natural frequencies, respectively.

1. Introduction

In recent years, microbeams made of functionally graded materials (FGMs) with high sensitivity and desired performances have been widely used in modern technology as a fundamental building block; for example in micro/nano electromechanical systems (MEMS/NEMS) [1,2], atomic force microscopes (AFMs) [3] and sensors [4,5]. The FGMs as a new class of advanced materials have superior properties with respect to the conventional composite materials such as thermal protection from ablation and elimination of stress concentration due to the smooth and continuous variations of their physical properties in spatial domain [6].

In some applications, one may prefer to use the varying section microbeams to generate the microbeams with optimal structural and other physical performance or due to the design restriction [7]. In addition, owing to the attached equipments, the microbeams may carry additional microparticles; for example, integrated complementary metal oxide semiconductor (CMOS)-MEMS free–free beam resonator arrays [8], AFM probes [3], and micro/nano scale sensors such as microaccelerometers [9]. The last example has been modeled as a cantilever microbeam with an end lamped mass. On the other hand, the micro-structural components made of FGMs may be under severe thermal environments; for example, a thin-film platinum flexible temperature sensor, which can potentially be used to characterize the thermo-ablation process of tumors, is operational for the temperature rise up to 400 °C [4]. But, it is well known that the temperature rise reduces the stiffness of the microbeams due to softening of the materials, and also, induces the thermal stresses, which consequently change the vibration behavior of these micro-structural elements. Hence, the change in vibration characteristics of the microbeams due to their section variation, attached microparticles and the thermal effects is of great importance and should be considered for a high-quality design and manufacture.

It is well known that the size dependence of the material properties can not be explained based on the classical continuum mechanics theories. On the other hand, the atomistic simulation needs much time and becomes computationally expensive for analyzing microstructures with relatively large numbers of atoms. To overcome these limitations and to develop simple and computationally efficient theories, recently, higher-order (non-local) continuum theories that contain additional material constants such as Eringen's nonlocal elasticity theory [10], the couple and modified couple stress theories [11,12], and the strain gradient theory (SGT) [13,14] have been introduced and widely used to study the mechanical behavior of nano/micro sized structural elements; see for example [15–29].

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It has been shown that the couple stress theory (CST), which is a general form of the modified couple stress theory (MCST), underestimates the size effect because it only employs the rotation gradient and neglects the other gradients [30]. The SGT is the reformulated and extended version of the CST in which the second-order deformation tensor separated into the stretch gradient tensor and rotation gradient tensor. This leads to additional higher-order stress components compared to the CST. But, this theory needs five higher-order material length scale parameters to complete the corresponding constitutive relations of a linear elastic material. Due to the experimental difficulty associated in the evaluation of these parameters, Lam et al. [31] modified the SGT and reduced the number of higher-order material length scale parameters from five to three for isotropic linear elastic materials. This theory can be degenerated into the MCST by appropriate chosen of the length scale parameters. In the MSGT, the dilatation gradient, deviatoric stretch gradient and symmetric rotation gradient are considered. In addition, in spite of the nonlocal elasticity theory of Eringen, CST, MCST and the SGT, a new additional equilibrium equation to govern the behavior of higher-order stresses is introduced. Also, it is shown that the equations of motions and the related boundary conditions (including the higher-order boundary conditions) can be derived by using the variational principle [31], which is one of the drawbacks of the nonlocal elasticity theory of Eringen.

In recent years, the MSGT of Lam et al. [31] has been employed by some researchers to study the different aspects of microbeams with the classical boundary conditions. In this regards, the MSGT in conjunction with the Euler-Bernoulli beam theory, Timoshenko beam theory or higher-order shear deformation beam theory have been used to investigate the static and vibration characteristics of microbeams; see for example Refs. [20–23,32–41]. To the best of authors' knowledge, only limited research works are available in the open literature which concerned with the vibration analysis of varying section microbeams [7,27,28,40]. Akgöz and Civalek [7] and Shafiei et al. [27] investigated the linear and nonlinear free vibration behavior of axially functionally graded tapered microbeams based on the MCST in conjunction with the Euler-Bernoulli beam theory, respectively. In another work, Shafiei et al. [28] studied the size dependent vibration behavior of the rotating non-uniform FG microbeams. They formulated the problem using the MCST together with both Timoshenko and Euler–Bernoulli beam theories and solved the equations of motion subjected to the related boundary conditions by employing the generalized differential quadrature element method (GDQEM). Zeighampour and Tadi Beni [40] presented the free vibration of axially functionally graded variable diameter microbeams by using the MSGT in combination with Euler-Bernoulli beam theory. They used the differential quadrature method to solve the governing equations. In all of these interesting works, the axially functionally graded tapered microbeams were analyzed.

The literature survey reveals that the free vibration behavior of the variable section FG microbeams carrying microparticles and under thermal environment has not been studied so far. Due to practical and theoretical importance of this issue, it is investigated in the present paper. For this purpose, the eigenfrequency equations of the microbeams are derived based on the MSGT together with the Timoshenko beam theory by using the Chebyshev–Ritz method. The material properties are assumed to be temperature-dependent and vary in the microbeam thickness direction. In addition, the influences of the initial thermal stresses are taking into account. The Chebyshev polynomials in conjunction with suitable boundary functions are used as admissible functions which enable one to study the influences of different boundary conditions on the vibration behavior of microbeams. The present approach is validated by performing the convergence study, and also comparing the results in the limit cases with those available in the open literature. Then, a detailed parametric study is carried out to show the influences of the material length scale parameter, material property gradient index, the ratio of microparticle masses to microbeam mass, and the taper ratio on the vibration behavior of microbeams. Some comparisons studies between the results of the different theories are also arranged.

2. Mathematical modeling

The FG microbeams under consideration have the length L, constant thickness h, and the width $b(x)$ which varies linearly along the x-axis from b_0 at $x=0$ to $b_1(\leq b_0)$ at $x=L$ (see Fig. 1). The microbeams carry two microparticles, one with mass m_1 at an arbitrary location on the microbeams at $x=L_1$, and the other with mass m_2 at $x=L$ as shown in Fig. 1. A Cartesian coordinate system with coordinate variables x, y, and z is used to locate the material points of the microbeams. It is assumed that the microbeams to be composed of two different phases (metal and ceramic phases), and the volume fractions of the material phases are assumed to vary continuously in the thickness direction. In the following, the basic governing equations and the solution procedure are presented.

2.1. Temperature-dependent FG microbeams relations

It is assumed that the material distribution in the thickness direction follows a power law as,

$$V_c=\left(\frac{z}{h}+\frac{1}{2}\right)^n,\quad V_m=1-V_c \tag{1a,b}$$

where V_m and V_c are the volume fraction of the metal and ceramic phases, respectively. Hereafter, the subscripts c and m are used to denote the material parameters of the ceramic and metal phases, respectively. It is observed from Eq. (1) that the upper surface of the microbeam (i.e., $z=h/2$) is the ceramic rich while its lower surface (i.e., $z=-h/2$) is the metal rich.

The effective material properties of the FGMs can be estimated suing different approaches. Among them, the rule of mixture is

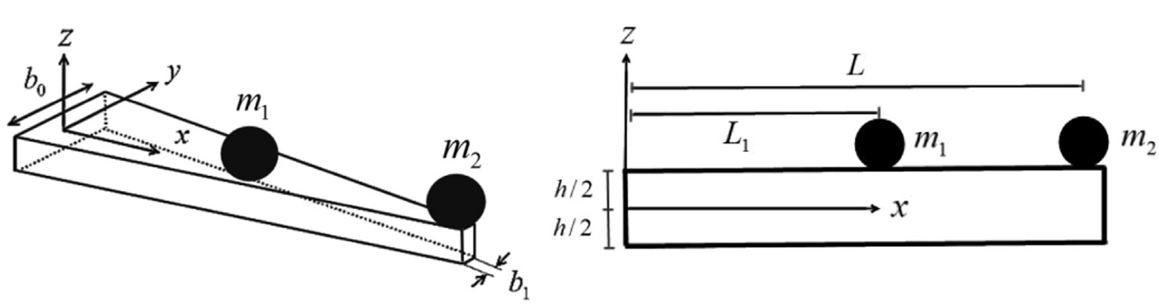

Fig. 1. Geometry of variable section microbeams with end and intermediate concentrated masses.