Thin-Walled Structures 107 (2016) 657–670

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Free vibrational characteristics of rotating joined cylindrical-conical shells

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ARTICLE INFO

Article history:
Received 24 November 2015
Received in revised form
5 July 2016
Accepted 12 July 2016
Available online 25 August 2016

Keywords:
Rotating shell
Joined cylindrical-conical shell
Free vibration
Forward wave
Backward wave

ABSTRACT

The aim of this study is to investigate the free vibration of a rotating joined cylindrical-conical shell in the presence of the centrifugal and Coriolis forces as well as the initial hoop tension resulting from rotation of the shell. Based on the equations of general rotating shells of revolution, the equations of motion of rotating cylindrical and rotating conical shells are derived, and then these governing equations were solved by employing the power series method. For validation and comparison purposes, the results of the special cases of the studied problem were compared with the results reported in the literatures. Another comparison study was also conducted by the results of finite element model of a rotating cylindrical-conical shell. It is observed that the presented approach predicts the frequencies of forward and backward waves with good accuracy. In addition, the effects of different parameters such as rotation speed, cone angle, circumferential wave number, length to radius ratio, and shell thickness on the frequencies of forward and backward waves were investigated.

1. Introduction

Due to their high strength-to-weight ratio, rotating cylindrical shells are extensively used in civil, mechanical and aerospace industries. The rotor systems of gas turbine engines, rotating satellite structures and the centrifugal separators in pharmaceutical industry are some typical applications of these types of shells.

Bryan [1] conducted the first study on rotating cylindrical shells in 1890. He investigated the vibrations of cylindrical shells by employing the rotating ring analysis and discovered the phenomenon of traveling-modes. In 1964, DiTaranto and Lessen [2] explored the effects of the Coriolis and centrifugal forces in infinitely long cylindrical shells. Hua and Lam [3] used the generalized differential quadrature (GDQ) method to study the effects of boundary conditions on the frequency characteristics of a thin rotating cylindrical shell. Wang et al. [4] have carried out a more recent work in this field, in which, by applying the Donnel's shell theory and using different multi-dimensional models, they considered the Coriolis force and damping effects in the equations of motion. By applying the harmonic reproducing kernel particle (HRKP) method, Liew et al. [5] studied the free vibrations of rotating cylindrical shells and explored the effects of different boundary conditions on the frequency characteristics.

Sun et al. [6] studied the vibration characteristics of rotating cylindrical shells by using the Sanders' shell equations and the Fourier series expansion method. In another paper, Sun et al. [7] studied the vibrations of rotating cylindrical shells with arbitrary edges by using the characteristic orthogonal polynomials and the Rayleigh-Ritz approach. In their investigation, they applied the Sanders' shell theory, and used elastic springs to simulate the arbitrary elastic supports.

Also, Song et al. [8] investigated the free vibrations of rotating cylindrical shells with elastic boundary conditions by applying the Donnel's shell theory and the Rayleigh-Ritz method. They derived the equations of motion for rotating cylindrical shells and explored the effect of elastic spring stiffness on the frequencies of these shells.

A few research activities have been conducted on the vibration of rotating conical shells. Based on the equations presented by Chen [9], Lam and Hua [10,11] derived the governing equations for isotropic rotating conical shells with simply-support boundary conditions and examined the effects of parameters like cone angle and geometric properties on the frequency characteristics. Using the same approach, Lam and Hua [12] and Hua [13] explored the effects of various parameters (e.g., boundary conditions, rotation speed and material orthotropy) on the frequency characteristics of orthotropic conical shells. The method presented by Chen [9] was also employed by Hua [14], Hua and Lam [15] and Ng et al. [16] to investigate the impact of different parameters on the frequency characteristics of composite laminated conical shells. Civalek [17] used the same equations to analyze the vibrations of isotropic rotating conical shells. He applied the discrete singular

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http://dx.doi.org/10.1016/j.tws.2016.07.009
0263-8231/

convolution (DSC) method to solve these equations and obtained frequency characteristics of the forward modes at different rotation speeds and boundary conditions.

Despite the extensive use of the rotating cylindrical-conical shells in various engineering fields especially in aerospace industries, no paper has been published on the free vibrations of these types of shells. In this paper, the free vibrations of a rotating cylindrical-conical shell is investigated. First, by using the general equations of rotating shells, the governing equations of rotating cylindrical and rotating conical shells are derived. Then, these governing equations are solved by employing the power series method and considering a displacement field which is harmonic with respect to time and the circumferential coordinate. The boundary conditions are written for the two ends of the cylindrical-conical shell, and the continuity conditions are considered for the interface of the two cylindrical and conical sections of the shell; and then by solving the eigenvalue problem, the natural frequencies of forward and backward waves are obtained.

2. Theoretical formulation

2.1. Governing equations for general rotating shells of revolution

The linear approximation method [18] is used to analyze the vibrations of general shells of revolution. In this approach, the nonlinear shell problem is approximated by two linear problems. Linear shell methods could be applied to solve each of these problems. By employing this method and considering large deformations and the Coriolis acceleration, Chen [9] presented the motion equations of rotating shells in 1993. Consider a shell (Fig. 1(a)) which is rotating with a constant velocity Ω about its symmetrical and longitudinal axis. The considered curvilinear coordinate system has been illustrated in Fig. 1(b). e_1, e_2, and e_ξ are the unit vectors in this coordinate system. Subscript ξ indicates the normal direction. Also, α and β denote the meridian and parallel coordinates, respectively.

The equations of motion for the general shells of revolution are initially expressed in vector form. Then by applying the method of linear approximation, these equations are divided into two groups of equations: basis state equations, and additional state equations [9].

Each set of the basic and additional states equations comprise six equations, which are presented in Appendices A and B, respectively. The basic state equations are derived from the first order terms of the main equation. These equations, which are independent of time, are known as the equilibrium equations resulting from centripetal forces. In the equations given in Appendix A, A and B denote the Lamé parameters. ρ, h and r are the shell density, shell thickness, and the rotation radius, respectively. Subscript I denotes the basic state. Also, N_{Iij}, Q_{Iij}, and M_{Iij} ($i, j = \alpha, \beta$) are the force, shear force, and bending moment resultants in the basic state, respectively. R_α is the radius of curvature of the meridian (α curve), and R_β is the radius of curvature of the normal section, tangential to the horizontal planar circle (β curve).

In the additional state, the equations are derived from the second order terms of the main equation. These equations, which are time-dependent, characterize the equations of motion and include the Coriolis forces and the effects of large deformations caused by the large centrifugal force. In the equations presented in Appendix B, u, v, and w are the mid-surface displacements of the shell along the meridional, circumferential, and normal directions, respectively. Subscript II indicates the additional state, and N_{IIij}, Q_{IIij}, and M_{IIij} ($i, j = \alpha, \beta$) denote the force, shear force, and bending moment resultants in the additional state. Also, ε_α, ε_β, $\varepsilon_{\alpha\beta}$, γ_α, γ_β are the shell strains and θ_α and θ_β are the rotations of the normal to the middle surface during deformation about the β and α axes, respectively which have been introduced in Appendix C [19].

2.2. Governing equations for rotating cylindrical shells

Consider the isotropic cylindrical shell shown in Fig. 2, which rotates with velocity Ω about its axis of symmetry. x is the longitudinal coordinate, θ is the circumferential coordinate, and z is the radial coordinate. The length, radius, and thickness of the cylinder are denoted by L^o, R^o, and h^o, respectively (superscript 'o' indicates a cylindrical shell). The shell's mid-surface displacements, along the x, θ, and z directions are indicated by u^o, v^o, and w^o, respectively.

2.2.1. Basic state equations for rotating cylindrical shells

In a cylindrical shell, parameters α, β, A, B, R_α, and R_β (presented in Appendices A and B) are defined as Eq. (1) [19].

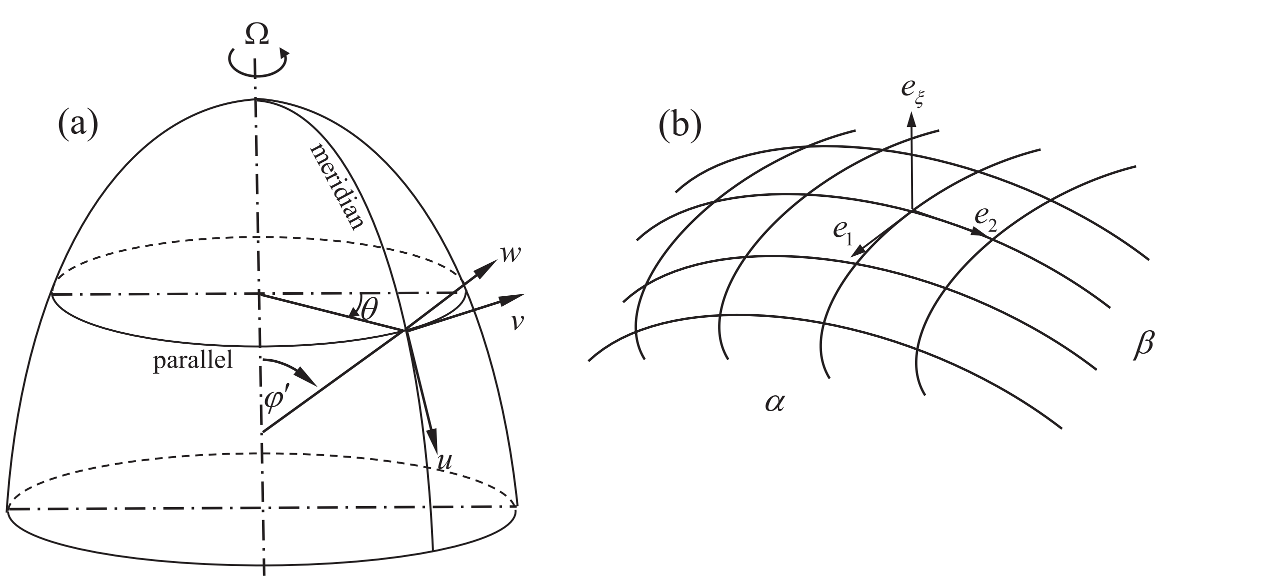

Fig. 1. A rotating shell in the curvilinear coordinate system.