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Free vibration study of sandwich plates using a family of novel shear deformable dynamic stiffness elements: limitations and comparison with the finite element solutions

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ABSTRACT

In this paper, the dynamic stiffness matrix of a completely free rectangular multi-layer plate element based on Reddy's higher-order shear deformation theory is derived. The reduction of the proposed model to the first-order shear deformation theory-based formulation is presented. Three coupled Euler-Lagrange equations of motion have been transformed into two uncoupled equations introducing a boundary layer function. The proposed model enables free transverse vibration analysis of rectangular multi-layer plates with (transversely) isotropic layers having arbitrary combinations of boundary conditions.

The influence of transverse shear deformation is discussed along with the applicability of two shear deformable dynamic stiffness elements. Moreover, the influence of the boundary conditions on the free vibration characteristics of sandwich panels has been discussed. The natural frequencies obtained using different dynamic stiffness multi-layer plate elements have been validated against the solutions from the commercial software Abaqus and the previously verified numerical solutions using layered finite elements. The limitations of the model regarding the differences between material properties of the face and core layers within a sandwich plate are highlighted. The influence of face-to-core thickness ratio on natural frequencies is illustrated, while a variety of new results is provided as a benchmark for future investigations.

1. Introduction

Sandwich panels are multi-layer structural members composed of two stiff faces and the core. They provide high stiffness to weight ratio which makes them applicable in different engineering fields. Sandwich panels are usually applied in civil engineering as light roof and wall panels to provide thermal insulation of buildings. Because of their position in the structure they are often exposed to dynamic loads such as turbulent wind loading. Consequently, adequate computational models capable to accurately predict the structural behavior of such structures in the dynamic loading environments are required.

The dynamic response of sandwich panels can be predicted using variety of solution strategies based on different plate theories. For thick plates, as well as for the multi-layered structures with considerable differences in the material properties between the adjacent layers, the classical plate theory (CPT) [1] based on Kirchhoff's hypothesis is not completely adequate because the transverse shear deformation is neglected. Consequently, the natural frequencies are over predicted, especially for higher modes of vibration. In the first-order shear deformation theory (FSDT), based on Mindlin-Reissner assumptions, the transverse shear effects are accounted for by means of the shear correction factors [2]. In Reddy's higher-order shear deformation theory (HSDT) [3] a cross-sectional warping is taken into account with a cubic approximation of the displacement field. The aforementioned theories are referred to as equivalent-single-layer (ESL) theories, which consider the multi-layer structure as a single homogeneous layer by taking into account different material and geometrical properties of the layers. Because the number of exact 3D elasticity solutions of multi-layer plates is generally limited [4,5], the finite element methods (FEM) have often been adopted to obtain the numerical solutions of the dynamic problems [6–11]. For example, the free vibration analysis of multi-layer panels using the HSDT is investigated in [9,12,13], while the comparison of different ESL plate theories is given by Reddy [14] and Chitnis et al. [15]. For an extensive overview of the ESL theories, interested readers are

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referred to the monographs by Staab [16], Reddy [17] and Carrera [18], amongst others.

To overcome the problems that may arise due to the simplifications associated with the classical plate kinematics in the ESL theories, the generalized layerwise plate theory (GLPT) of Reddy [19] is used to improve the representation of the kinematics of the layered structure. This theory is capable of representing the motion of each layer individually, leading to the cross-sectional warping, and also allows the independent interpolation of in-plane and out-of-plane displacement components. In the layerwise approach, it is assumed that C^0-continuity through the thickness of the plate is satisfied. The layered finite element based on the GLPT is proposed by Reddy et al. [20] and successfully applied by Ćetković and Vuksanović [21] as well as Marjanović and Vuksanović [22] in the free vibration analysis of intact and damaged laminated composite and sandwich plates.

In the free vibration analysis, the dynamic stiffness method (DSM) [23–26] has been used to obtain more accurate and reliable results in comparison with the conventional FEM. The DSM uses a unique element matrix (dynamic stiffness matrix) containing both stiffness and mass properties of the structure. The selection of the DSM for solving the free vibration problem is motivated by the fact that only one dynamic stiffness element per structural member with constant material and geometrical properties can be used to accurately represent its dynamic behavior at any frequency.

First applications of the dynamic stiffness method based on the classical plate theory are given in the works of Wittrick and Williams [27,28]. In the series of investigations Boscolo and Banerjee [29–32] and Fazzolari et al. [33] derived the exact dynamic stiffness matrices for isotropic and composite plates having two edges simply supported (Levy-type plates), based on the FSDT, HSDT and layerwise plate theories. However, the main lack in investigations [29–33] was the inapplicability of the proposed methods for plates having arbitrary combinations of boundary conditions. This issue can be overcome using the Projection method, which is presented in the works of Kevorkian & Pascal [34] and Casimir et al. [35]. The dynamic stiffness method for the free vibration analysis of rectangular orthotropic plates having arbitrary boundary conditions, based on the classical plate theory (CPT), has been presented in work of Papkov & Banerjee [36]. Banerjee et al. [37] introduced the method for the general case of rectangular plate based on the CPT. Moreover, Nefovska-Danilovic and Petronijevic [38] developed the dynamic stiffness element for the in-plane free and forced vibrations of plate assemblies having arbitrary boundary conditions, while Kolarevic et al. [39,40] presented the dynamic stiffness matrices for a completely free rectangular isotropic plate based on shear deformable (both FSDT and HSDT) kinematics. Finally, Liu and Banerjee [41–43] gave the comprehensive contribution in free flexural vibration analysis of isotropic and orthotropic composite plate assemblies, based on the classical plate theory. The solutions [34–43] do not have any restrictions regarding the boundary conditions, making the dynamic stiffness method applicable in a variety of practical applications.

In this paper, the previous work of Kolarevic et al. [40] is extended to derive the dynamic stiffness matrix of a completely free rectangular multi-layer plate element based on the HSD Theory. Three coupled Euler-Lagrange equations of motion have been transformed into two uncoupled equations of motion using a boundary layer function [44–46]. The proposed method enables free transverse vibration analysis of rectangular multi-layer plates with transversely isotropic layers, having arbitrary combinations of boundary conditions. The transverse displacement of the plate element has been split into four symmetry contributions according to the superposition method [47]. After the development of the dynamic stiffness matrices for each symmetry contribution using the Projection method [34,35], the dynamic stiffness matrix of completely free HSDT plate element has been derived superposing the dynamic stiffness matrices of the four symmetry contributions. In addition, the reduction of the proposed method to the FSDT-based formulation is presented, thus the FSDT-based dynamic stiffness element of the multi-layer plate is easily derived. The natural frequencies and the corresponding mode shapes obtained using different dynamic stiffness multi-layer plate elements have been validated against the solutions from the commercial software Abaqus [48] and the previously verified numerical results based on the GLPT, using the so-called “layered finite elements” [21,22].

The main goal of the paper is to illustrate the effects of transverse shear deformation in the context of the applicability of the HSDT- and the FSDT-based dynamic stiffness elements in the free vibration analysis of sandwich panels. The limitations of two computational models based on the DSM (regarding the differences between the material properties of the face and core layers within a sandwich plate) are highlighted. The influence of the face-to-core thickness ratio on the natural frequencies is illustrated, while a variety of new results is provided as a benchmark for future investigations. Finally, the effect of boundary conditions on the free vibration characteristics of sandwich panels has been discussed.

The paper is organized in the following steps. Section 2 summarizes the formulation of the HSDT for the multi-layer structures. The derivation of the HSDT-based dynamic stiffness element and the solution procedures are given in detail in Section 3. The reduction of the HSDT model to the FSDT one is provided in Section 4. The verification and applicability of the proposed method as well as a variety of new benchmark examples are presented in Section 5. Finally, concluding remarks on the proposed model and its further improvement are provided.

2. Formulation of the multi-layer HSDT dynamic stiffness element

2.1. Basic assumptions

The geometry of rectangular sandwich plate composed of isotropic layers is presented in Fig. 1. The overall plate thickness is denoted as h, while the thickness of the k^{th} layer is h_k. The origin of the Cartesian coordinate system is located in the mid-plane of the plate, with a z-axis pointing upwards. The assumptions and restrictions introduced in the derivation of the model are:

1. All layers are perfectly bonded together.
2. The material of each layer is homogeneous, isotropic and linearly elastic.
3. Linear kinematics (small strains and small rotations) are assumed.
4. Inextensibility of the transverse normal is imposed. Cross-sectional warping is accounted.

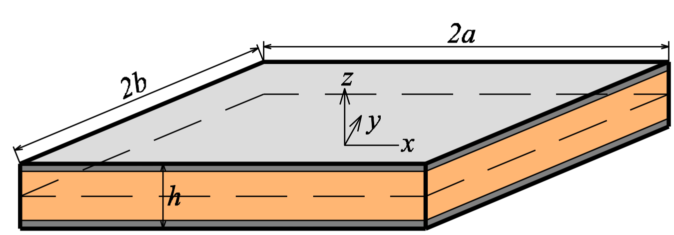

Fig. 1. Geometry of sandwich plate.