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Analytical model for buckling analysis of the plates under patch and concentrated loads

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ABSTRACT

The problem of elastic stability of rectangular plates with different boundary conditions under wheel and patch loading is analyzed using Ritz energy technique.

Basic analytical model with two specific types of boundary conditions, designed to simulate standard experimental sample, is tested. High quality of presented analytical results, proved through comparison with data obtained by numerical methods, is due to introduction of the exact stress functions of Mathieu's theory of elasticity and adequate deflection functions.

Hence, the conditions for analyzing the advanced models with different levels of complexity and for realization of comprehensive parametric study are created for these load types.

1. Introduction

Buckling problem of high steel girders under variable external loads is still very interesting topic in steel structures. Presently available literature abounds with data regarding this problem, but mostly obtained by numerical or experimental methods. Analytical approach has been avoided mostly because of unknown stress distribution.

In the series of papers based on Mathieu's method from 1890 [1], Baker, Pavlović and Tahan [2] and later Liu [3] and Mijušković [4] developed very precise approach for exact stress function determination for main case of rectangular plate under arbitrary external load. Existence of such solutions created the basis for the analysis of very complex stability problems in real steel structures.

Analytical approach to critical load determination, based on exact stress functions implementation, is verified for relatively simple case of plate under (DEA) uni and biaxial compression [3–5]. In this paper the next step is introduced through a significantly complicated problem of the plate under concentrated force(s) and locally distributed load (patch loading) applied on the upper flange of steel I-girder.

In the aim of testing developed analytical approach, the case of patch loading is deliberately chosen as one of the most common and most challenging in real steel structures. Reason more for the selection of this load is the statement, very often found in literature, that this particular problem, especially for plates with different boundary conditions, can not be accurately tackled by existing analytical approaches. To be more specific, general opinion is that analytically obtained buckling load underestimate real critical values as a consequence of not only inability to use exact stress distribution within plate, but also disregarding several important factors whose implementation in existing analytical methods of 2D elasticity theory is considered very difficult. However, now, with possibility of applying the exact two-dimensional elasticity solutions for plates under arbitrary external load, many parameters, especially regarding flange effects (flexural and torsional rigidity of the flange, different degrees of elastic edge restraint, load dispersion by the flange thickness and, as the most important, relaxation effects of shear stress at the flange-web junction) may be included in the analytical model. Said that, the case of patch loading is ideal for analytical approach demonstration because it enables gradual upgrading of the basic mathematical model, presented in this paper. Hence, the main goal of this research is gradual implementation of all (or most of all) aforementioned parameters in order to define realistic models and reach the real values of the critical loads. In future papers, the case of patch loading will be analyzed by using different mathematical models which are describing the mentioned problem with different levels of complexity.

The simplest and the most often structural solution for the problem of intensive concentrated load has been vertical

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Nomenclature

a Plate width (measured along the x-axis)
A_o, A_n, A_m Fourier-series coefficients for arbitrary external loadings $f(y)$, $f(x)$
b Plate length (measured along the y-axis)
B_n, B_0 Unknown group of coefficients defined by Eqs. (13) and (14)
$c=l_2/a$ Ratio of spacing (loaded area) between point loads to width of plate
$D=Et^3/12(1-\nu^2)$ Flexural rigidity of the plate
$e(x)=\sinh(x)$ Abbreviation
$E(x)=\cosh(x)$ Abbreviation
E Young's modulus
$f(x)$, $f(y)$ Functions defining the applied load on the edges $y=\pm b/2$ and $x=\pm a/2$
F, $(F=F_1+F_2)$ Function introduced to satisfy Eq. (6)
G_m Unknown group of coefficients defined by Eq. (12a)
H_n Unknown group of coefficients defined by Eq. (12b)
l_1 Load length on the edges $x=\pm a/2$
l_2 Spacing between point loads
K Buckling coefficients for the case of distributed load
K_T Buckling coefficients for the case of concentrated force
N_1, N_2 Analytical solutions for direct stresses along x and y axis respectively
P Concentrated (point) load
t Thickness of the web
T_3 Analytical solution for shear stress in the plane x-y
u, v Displacements along the x and y directions respectively
U Strain energy due to bending
V Potential energy associated with the work done by external loads
w Out-of-plane deflection function
W_{mn} Coefficients for deflection function
$\alpha=(\lambda+2\mu)/\mu$ Constant defined in terms of Lame's parameters
β_m Unknown group of coefficients defined by Eqs. (13) and (14)
$\gamma=l_1/a$ Patch-load length ratio
Δ Laplase's operator
$\varepsilon=\mu/(\lambda+\mu)$ Constant defined in terms of Lame' parameters
Z_i Functions in non-dimensional form defined by Eqs. (16)
$\lambda=\nu E/(1+\nu)(1-2\nu)$ Lame's parameter
Λi Functions in non-dimensional form defined by Eqs. (15)
$\mu=E/2(1+\nu)$ Lame's parameter
$\boldsymbol{\nu}$, $(\boldsymbol{\nu}=\boldsymbol{\nu_1}+\boldsymbol{\nu_2})$ Volumetric dilatation
ν Poisson's ratio
Π Total potential energy of the system
$\sigma(x)=e(x)-x/E(x)$ Abbreviation
σ_x, σ_y Direct stresses
τ_{xy} Shear stress in the plane x–y
$\tau(x)=E(x)-x/e(x)$ Abbreviation
$\phi=a/b$ Aspect ratio of the plate
$\chi(x)=E(x)/\sigma(x)$ Abbreviation
$\Psi(x)=e(x)/\tau(x)$ Abbreviation

(transverse) stiffener underneath the load with the task to partially take over vertical stresses and prevents possible web buckling. However, modern design of structures tends towards more daring and more economic solutions which result in omitting vertical stiffeners wherever possible. Besides, in case of moving loads (e.g. crane girder, launching of bridges etc.) it is not possible to secure each critical position of load by vertical stiffener. Therefore, it is completely justified, for standard plate aspect ratios, to analyze stability of patch loaded steel girders without additional vertical stiffeners (Fig. 1.1).

In order to define mathematical model for locally distributed load applied perpendicular to the upper flange, web of I or box girder is commonly treated as isolated plate with shorter edges (i.e. edges along transverse stiffeners above girder supports, Fig. 1.1) usually considered as simply supported. On the other side, longitudinal edges are elastically restrained to the certain point, depending on the rigidity of the flanges. Hence, two mathematical models, simulating two specific cases of light (with small resistance to rotation) and heavy (basically rotationally restrained, even clamped) flanges, are considered in this paper. Thus stability analysis of simply supported plate (SSSS) and plate with pair of simply supported and pair of clamped edges (CSCS) provides lower and upper bounds of critical buckling load for the selected external load case.

Initial, basic mathematical model chosen to represent buckling problems of plates under concentrated and locally distributed compression (Fig. 1.2) is built through superposition of two (DEA and DEB) fundamental load types and represents a typical experimental sample.

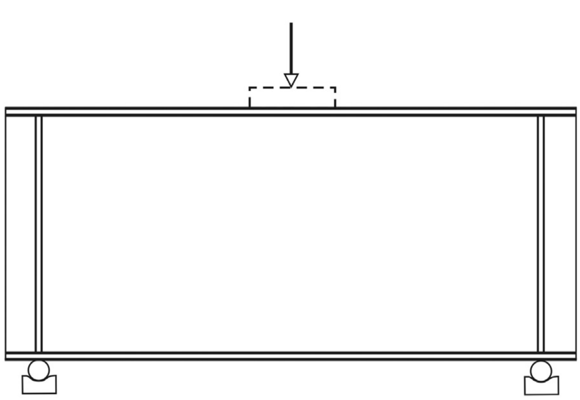

Fig. 1.1. Girder without transverse stiffeners.

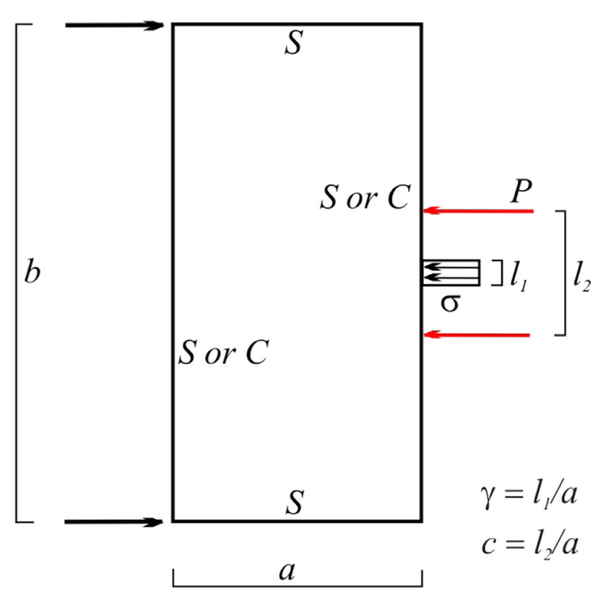

Fig. 1.2. Basic (initial) model for patch-loading analysis.