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Analytical research on dynamic buckling of thin cylindrical shells with thickness variation under axial pressure

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ABSTRACT

This paper focuses on the dynamic buckling of thin cylindrical shells with arbitrary axisymmetric thickness variation under time dependent axial pressure. Based on the derivation of stability and compatibility equations of variable thickness cylindrical shells under dynamic external pressure by Aksogan and Sofiyev, the corresponding stability and compatibility equations of thin cylindrical shells with arbitrary axisymmetric thickness variation under dynamic axial pressure are obtained and expressed in nondimensional form. Combining the small parameter perturbation method, Fourier series expansion and the Sachenkov–Baktieva method, analytical formulas of the critical buckling load of thin cylindrical shells with arbitrary axisymmetric thickness variation under axial pressure that varies as a power function of time are obtained. Two cases of thickness variation are introduced to research the critical dynamic buckling load with the present formulas. Effects of thickness variation parameters and loading speed of dynamic axial pressure on the critical buckling load are discussed. The method is also applied to determine the critical dynamic buckling load of thin cylindrical shells with a classical cosine form thickness variation. Results revel that the thickness variation and pressure parameters play a major role in dictating the buckling capacity of thin cylindrical shells under dynamic axial pressure.

1. Introduction

Cylindrical shells with thickness variation have gained more and more application in the oil, marine and aerospace fields in recent years [1–7]. Utilization of materials becomes more efficient and economic rationality is improved by using this structure. Dynamic buckling problems of cylindrical shells under axial pressure have been researched in previous literature.

Tamura and Babcock [8] investigated the dynamic stability of an imperfect circular cylindrical shell subject to a step loading in the axial direction. The critical loads were determined by numerical integration of the equation of motion and compared with the static case. Huyan and Simitses [9] researched the dynamic stability of circular metallic and laminated cylindrical shells subjected to axial compression with geometrically imperfection. The dynamic critical loads were determined by the method of finite element and the equations of motion. Effects of axial pressure and structural imperfection on critical loads were discussed. An analytic solution for the stability behavior of cylindrical shells made of compositionally graded ceramic–metal materials under the axial compressive loads varying as a power function of time was provided by Sofiyev [10]. Effects of the variations of loading parameters and power of time in the axial load expression on the critical parameters of the shell were elucidated. Bisagni [11] dealt with dynamic buckling of fiber composite shells under impulsive axial compression using finite element method and experiment tests. It showed that the dynamic buckling loads strongly depended on the load duration and initial geometric imperfections. The dynamic buckling of thin isotropic thermoviscoplastic cylindrical shells compressed with a uniform axial impact was investigated analytically and numerically by Wei et al. [12] and the axisymmetric and non-axisymmetric dynamic buckling phenomena of cylindrical shells subjected to impacts of axial load was presented by Xu et al. [13]. Sofiyev et al. [14] studied the dynamic stability of orthotropic cylindrical shells with non-homogenous material properties under axial compressive load varying as a parabolic function of time. Expressions of the critical dynamic axial load and dynamic factor had been derived and effects of the axial loading parameter on the critical parameters had been researched. Semi-analytical solution for critical parameter values of dynamic axial compressed composite orthotropic cylindrical shell was obtained by Sofiyev et al. [15]. Huang and Han [16] researched the nonlinear dynamic buckling of functionally graded cylindrical shells subjected to axial load that varies linearly with regard to

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time. Effects of the loading speed, dimension parameter and initial geometrical imperfection on critical condition were discussed.

Besides the cylindrical shells under dynamic axial pressure, typical shell structures with variable thickness under dynamic external pressure have also attracted attention of some researchers. Aksogan and Sofiyev [17] and Sofiyev et al. [18] combined the Galerkin and Rize type variational method to obtain analytical solution of the critical static and dynamic loads, the corresponding wave numbers and dynamic factor of cylindrical shells with variable thickness under dynamic external pressure. Based on these methods, Sofiyev [19] and Sofiyev and Aksogan [20] researched buckling of conical shells with variable thickness under dynamic external pressure and found analytical expressions of the critical static and dynamic loads, the corresponding wave numbers, dynamic factor and critical stress impulse. Results showed that variation of thickness, external pressure and semi-vertex angle had appreciable effects on the critical dynamic factors.

In addition, research on the buckling of cylindrical shells with non-uniform thickness subjected to static pressure had also been done by some researchers [21–29].

Review of the previous literature shows that researches are most about the cylindrical shells with constant thickness or made of composite materials under dynamic axial pressure, and cylindrical shells or conical shells with variable thickness under dynamic external pressure, which is perpendicular to the outside surface of the shells. Research on the cylindrical shells with thickness variation under dynamic axial pressure is quite limited. Thus we were prompted to present a theoretical method for obtaining analytical expressions of critical dynamic buckling load under axial pressure which takes on a power function of time, which can be applied to thin cylindrical shells with arbitrary axisymmetric thickness variation.

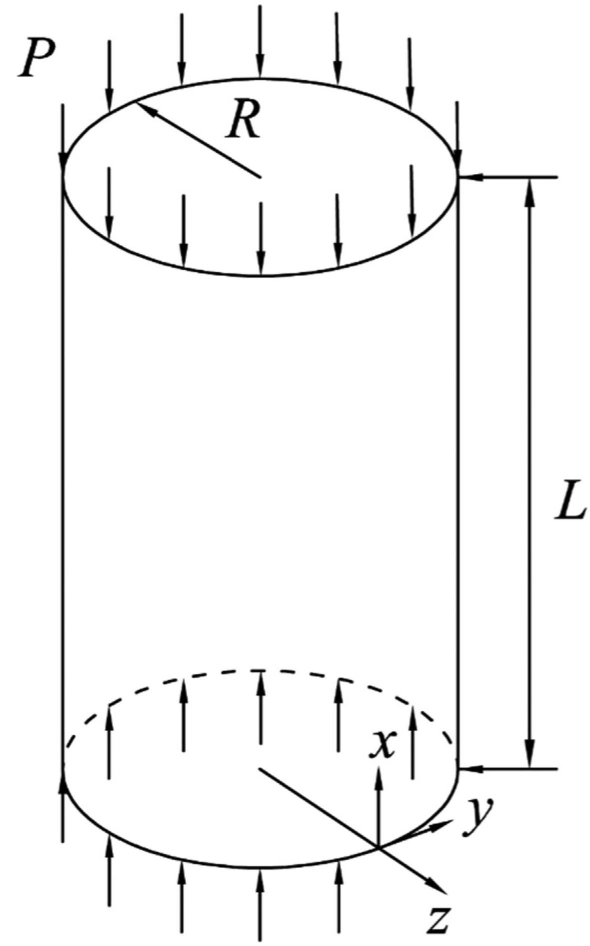

Fig. 1. Geometry and coordinate of the axially compressed cylindrical shell.

2. Fundamental equations

As shown in Fig. 1, a thin cylindrical shell of length L and radius R is subjected to dynamic axial compressive load varying as a power function of time, i.e.

$$P = P_0 t^{\alpha} \tag{1}$$

where P is the dynamic uniform pressure, P_0 the loading speed, t the time and α the positive whole number power which express the time dependence of axial pressure.

The linear strain–displacement relations of a thin cylindrical shell can be expressed as

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} - \frac{w}{R} \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$
$$\kappa_x = -\frac{\partial^2 w}{\partial x^2} \qquad \kappa_y = -\frac{\partial^2 w}{\partial y^2} \qquad \kappa_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \tag{2}$$

where u, v, w are displacements of the shell's middle surface along axial, circumferential and radial (positive inward) directions respectively. ε_x, ε_y, γ_{xy} are the strain components. κ_x, κ_y, κ_{xy} are curvature components of the middle plane.

The following compatibility equation can be obtained by applying simple algebraic operation to Eq. (2)

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} \tag{3}$$

Based on the Donnell simplified shell theory, the stability equations of a thin cylindrical shell under dynamic axial pressure are given as follows

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2} \tag{4}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \tag{5}$$

$$\frac{\partial N_{xz}}{\partial x} + \frac{\partial N_{yz}}{\partial y} + \frac{N_y}{R} + N_x^0 \frac{\partial^2 w}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} + N_y^0 \frac{\partial^2 w}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{6}$$

where N_x, N_y, N_{xy} are membrane force components, N_{xz}, N_{yz} are bending force components, N_x^0, N_y^0, N_{xy}^0 are membrane force components in the initial stability state, and ρ is material density. For the uniform axial pressure as shown in Eq. (1), the membrane force components satisfy $N_y^0 = N_{xy}^0 = 0$, $N_x^0 = -hP_0 t^{\alpha}$.

The moment components are given as follows

$$M_x = D\left(\kappa_x + \upsilon \kappa_y\right) \qquad M_y = D\left(\kappa_y + \upsilon \kappa_x\right) \qquad M_{xy} = D\,(1 - \upsilon)\kappa_{xy} \tag{7}$$

where $D = Et^3/12(1 - \upsilon^2)$ is the flexural rigidity and υ is the Poisson ratio.

The moment equilibrium equations are given by

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - N_{xz} = 0$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - N_{yz} = 0 \tag{8}$$