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Elastic buckling of a sandwich beam with variable mechanical properties of the core

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ABSTRACT

This paper deals with the analytical and numerical studies of elastic buckling of a three-layered beam with metal foam core. Mechanical properties of the core are variable along the *z*-axis. There are two schemes of displacement of the faces and core of the beam: a broken line hypothesis and a non-linear hypothesis. The mathematical models for both types of displacements are presented. The governing differential equations of the sandwich beam are derived. Numerical analysis of sandwich beams is conducted in ANSYS environment. The finite elements analysis has been performed using a linear elastic buckling model. The analysis with constant and variable Young's modulus of the core of the beam is carried out. The values of the critical load obtained by the analytical and numerical (FEM) methods are compared.

1. Introduction

Sandwich structures are being investigated since the mid-20th century. The foundations of the theory of that kind of structures are presented in the literature by Libove and Butdorf [1], Plantema [2], and Reissner [3].

The presented paper is devoted to the three-layered beam with an aluminum foam core. Manufacture, characterization and application of cellular metals and metal foams are described by Banhart [4]. Bart-Smith et al. [5] studied the influence of imperfections on the performance of metal foams. The size effect of cells was investigated by Kesler and Gibson [6]. Magnucka-Blandzi and Magnucki [7] described the effective design of sandwich beams with a metal foam core. Magnucka-Blandzi [8] considered also a non-linear hypothesis of core deformation of elastic sandwich beam cross-section.

Recently, the problem of fatigue and stability of sandwich structures was investigated by Bauchau and Craig [9] and Vinson [10]. Burman and Zenkert [11,12] described the fatigue of metal foam core. Hart [13] studied the fatigue and strength of sandwich beams with an aluminum alloy foam core. Magnucki and Szyc [14] and Magnucki et al. [15] investigated the strength, stability and deflection of sandwich structures with an aluminum foam core. Dynamic stability of a sandwich plate was studied by Dębowski et al. [16]. Modeling of layers beams was described by Bauchau and Craig [9], Magnucka-Blandzi [8,17], Magnucki et al. [18], and Magnucki and Szyc [14].

In the literature one can find a lot of descriptions of buckling of sandwich beams, columns and plates. Ding and Hou [19] considered general buckling of the mentioned constructions. Global and local bucklings of sandwich beams were investigated by Douville and Le Grognec [20], Huang and Kardomateas [21], Jasion et al. [22,23,24], Bazant and Beghini [25], Javidinejad [26], Magnucki and Stasiewicz [27], Magnucki [28]. Local buckling of faces was described, e.g., by Aviles and Carlsson [29], Jasion and Magnucki [30,31]. Theoretical and experimental studies of sandwich circular plate under pure bending were discussed by Magnucki et al. [32]. The influence of physical nonlinearity on buckling of sandwich beams was studied by Koissin et al. [33]. Buckling analysis of three-layer beams with double delaminations was described by Parlapali and Dongwei [34]. Kardomateas et al. [35], Kueh [36], Kwaśniewski and Ziółkowski [37], Leotonig et al. [38], Magnucki [28], Ostergaard [39] studied various cases of buckling of sandwich columns. Buckling of composite sandwich structures was described by Magnucka-Blandzi [40], Rivalant et al. [41], Teter and Kolakowski [42]. Numerical (FEM) investigations of buckling of sandwich structures were conducted by Bazant and Beghini [25], Dębowski et al. [16], Kwaśniewski and Ziółkowski [37], Pokharel and Mahendran [43], Veedu and Carlsson [44]. Laboratory tests of buckling of the mentioned structures were made by, Jasion et al. [22], Jeyakrishan [45], Magnucki [28], Magnucki and Szyc [14], Pokharel and Mahendran [43], Veedu and Carlsson [44].

In the literature one can also find studies of buckling of composite structures (e.g., Rivallant et al. [41], Teter and Kolakowski [42]) or honeycomb structures (e.g., Jeyakrishnan et al. [45]).

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As it was mentioned earlier, the subject of the present paper is elastic buckling of the three-layered beam with an aluminum foam core with variable mechanical properties. Two analytical and numerical (Finite Element Method) models are studied. The scheme of considered sandwich beam, its geometry, load and supports is shown in Fig. 1.

The length of the beam equals L. The total thickness of the beam is $H = t_c + 2t_f$, where t_c is the thickness of the core and t_f is the thickness of the faces. The width of the beam equals b. The beam carries a compressive axial force F_0. The layers of glue between the core and the faces are not taken under consideration. Also, the local phenomenon such as wrinkling of the faces is not taken into account.

The goal of this paper is to elaborate the mathematical model of the sandwich beam which includes isotropic faces and core and to investigate the influence of the core properties on the behavior of the structure. The formula describing the critical load corresponding to the elastic global buckling is looked for here. The model will be verified by the FE method.

2. Analytical studies of the sandwich beam

In the paper two mathematical models are presented. Both models are derived with linear mechanical properties of the material of the faces. The classical broken line hypothesis is used to describe the model of displacements (Fig. 2) in the first case. In the second case a non-linear hypothesis of displacements of the core is considered. In this case the non-linear hypothesis [8] describes the model of displacement taking into consideration the shift of a neutral line due to variable Young's modulus (Fig. 3).

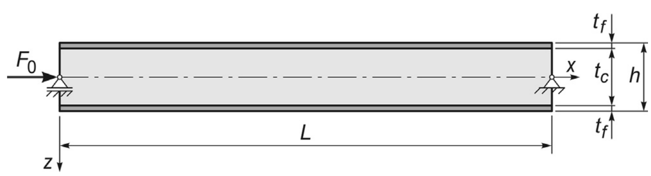

Fig. 1. The scheme of the three-layered beam.

Classical broken line hypothesis. The scheme of the sandwich beam before deformation is shown in Fig. 2. The beam deforms if the critical buckling load is reached. Based on the scheme presented in Fig. 2, longitudinal displacement $u(x, \zeta)$ of each layer of the beam after deformation is determined:

a) the upper sheet-face: $-(1/2 + x_1) \le \zeta \le -1/2$

$$u(x,\zeta) = -t_c\left[\zeta\frac{dw}{dx} - \psi(x)\right], \tag{1}$$

b) the lower sheet-face: $1/2 \le \zeta \le (1/2 + x_1)$

$$u(x,\zeta) = -t_c\left[\zeta\frac{dw}{dx} + \psi(x)\right], \tag{2}$$

c) the metal foam core: $-1/2 \le \zeta \le -1/2$

$$u(x,\zeta) = -t_c\zeta\left[\frac{dw}{dx} - 2\psi(x)\right], \tag{3}$$

where $\zeta = \frac{z}{t_c}$ is dimensionless coordinate, $w(x)$-deflection function, $\psi(x) = \frac{u_f(x)}{t_c}$ is dimensionless displacement, and $x_1 = \frac{t_f}{t_c}$ is the ratio of thicknesses.

Non-linear hypothesis. Taking into consideration the non-linear hypothesis of displacement one should know that the faces of the beam are made of an isotropic material with linear stress–strain relations.

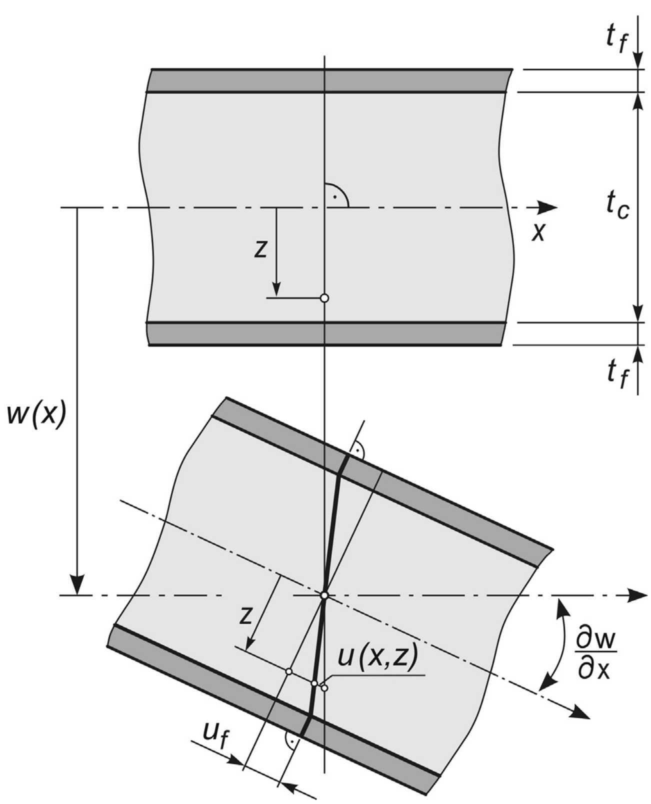

Fig. 2. Scheme of deformation of the plane cross-section of the sandwich beam–classical broken line hypothesis.

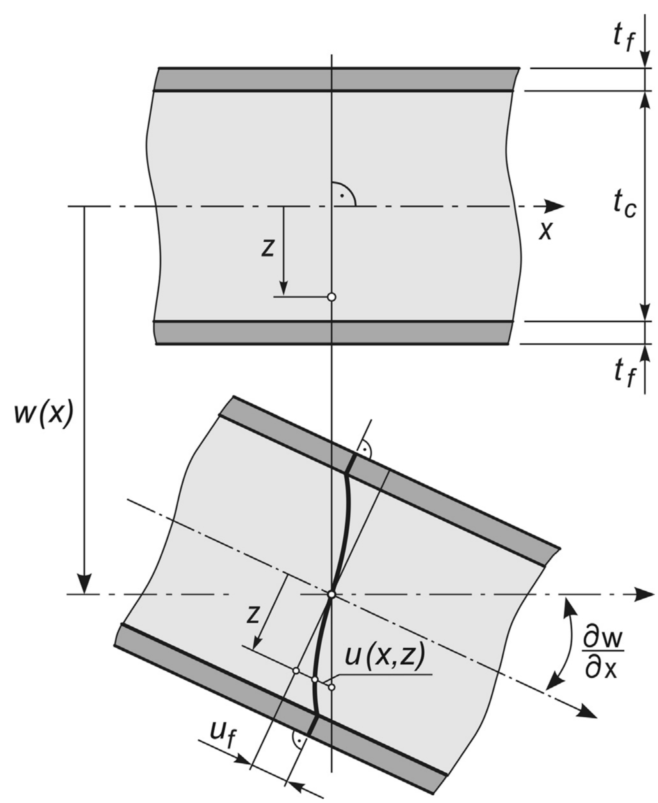

Fig. 3. Scheme of deformation of plane cross section–non-linear hypothesis [27].