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Optimal internal pressurisation of cylindrical shells for maximising their critical bending load

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ABSTRACT

The paper studies the influence of internal pressure on circular thin-walled pipes ($D/t>150$) subjected to pure bending. Both straight pipes and curved pipes are analysed. Both yield and buckling failures are considered. It is shown that internal pressure decreases the limiting load for yield but increases the limiting load for buckling.

The study is mainly FEA-based. A formula to predict critical moment given by linear buckling analysis is proposed. Comments on difference between linear and non-linear analysis results are given. It is shown that a pipe curvature opposite to the bending moment can increase the critical load. It is shown that cylindrical thin-walled shells have an optimal value of internal pressure to which limiting load for yield and critical buckling moment are equal, corresponding to an optimal use of material.

1. Introduction

Slender parts and thin-walled components are frequently used for limiting the structural weight. They are widely used in engineering and find many applications in aeronautical and aerospace field wherein lightness is key factor. However, such structures are naturally subjected to static stability problems under compression loads, i.e. buckling failure. Depending on their geometry, material and load conditions, buckling could be the most likely type of failure.

Buckling phenomenon has been widely investigated by several authors [1–3]. Some focused on thin cylindrical shells [4–6]. Specific studies have been conducted on curved pipes and showed that pipe curvature decreases the critical load [7]. Also the benefits in terms of buckling of internally pressurised cylindrical shells have long been known, in cases of axial loading [8], torsion [9] and bending [10,11].

The main novelty of this work is the exhibition of the fact that a cylindrical thin-walled shell under bending is characterised by an optimal value of internal pressure which maximises the critical applied bending moment. Such conditions correspond to a maximum exploitation of the structural potential of the shell.

The paper is organised as follows: Section 2 summarises the previous research on buckling of cylindrical thin-walled shells; Section 3 provides and an analytical treatise on limiting load for yield of pipes under bending and internal pressure; Section 4 describes the model used in FE analyses and presents the obtained results; Section 5 concludes the paper.

2. Background

The first studies of collapse of cylindrical thin-walled shells started analytically and focused on axial compressive loads. Tymoshenko and Gere [1] found that the critical stress for a long cylindrical shell simply supported at the ends is expressed by

$$\sigma_{cr}=\frac{Et}{r\sqrt{3(1-\nu^2)}} \tag{1}$$

where σ_{cr} is the critical stress, E is the Young Modulus, t is the wall thickness, r is the mean pipe radius and ν is the Poisson's ratio.

For cylindrical thin-walled shells bending moment and maximum stress are related as follows:

$$\sigma=\frac{M}{\pi r^2 t} \tag{2}$$

where σ is the maximum stress in the axial direction and M is the bending moment.

Assuming that a circular thin-walled pipe under pure bending buckles when the compressive stress reaches the value corresponding to buckling due to axial compressive load, the critical moment M_{cr} can be calculated by simply combining Eqs. (1) and (2). For

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a steel with $\nu = 0.3$ one obtains:

$$M_{cr} = \sigma_{cr}\pi r^2 t = 0.605\pi E r t^2 \tag{3}$$

Tymoshenko and Gere [1], Yudo and Yoshikawa [7], Brazier [4], Chwalla [12] and other authors conducted studies on circular pipe under bending and their results range over values in between 0.55 and 1.3 times the value expressed by Eq. (3). The reason for this variability is that Eq. (3) provides a reference value for critical moment but it cannot represent the real value for circular pipes since their behaviour and their strength is strongly depending on other parameters, especially diameter-thickness ratio D/t [13].

A formula to calculate the buckling moment M_p in plastic region was proposed [14]:

$$M_p = \left(1.05 - 0.0015\frac{D}{t}\right)\sigma_y D^2 t \tag{4}$$

where D is the pipe diameter and σ_y is the yield stress. This equation is widely accepted to be a good design criterion. Plastic buckling will not be taken into account in this paper since its occurring is usually due to a low value of yield stress and/or a low value of D/t ratio [3].

3. Limiting load for yield

Pure bending refers to a load condition wherein bending is the only load acting in a member. Despite the fact that there are no actual cases of structural members subjected to pure bending in reality, their study is relevant since it can provide a prediction of the type of failure and a limit allowable load for similar load condition.

A member subjected to bending will have a non-constant stress along the cross-section being partly under tension and partly under compression. Increasing the bending moment, the tensile part could reach yield whereas the compression part could buckle. Here limiting load for yield of a cylindrical internally pressurised thin-walled pipe is analytically studied.

In order to calculate the limiting load for yield of a cylindrical thin-walled shell subjected to internal pressure and bending moment, one has to take into account the stresses caused by these two different loads. For the sake of simplicity, it is assumed that the material is linear and isotropic and the cross-section does not change with loading.

Internal pressure will result in circumferential stress $\sigma_{\theta,p}$ and axial stress $\sigma_{z,p}$ where the subscript p indicates stress due to pressure.

$$\sigma_{\theta,p} = \frac{pD}{2t} \tag{5}$$

$$\sigma_{z,p} = \frac{pD}{4t} \tag{6}$$

Bending moment will result in a non-constant axial stress $\sigma_{z,m}$ along a cross-section where the subscript m indicates stress due to moment.

$$\sigma_{z,m} = \frac{M}{\pi r^2 t} \tag{7}$$

When both the loads – pressure and moment – are acting, the total stresses are

$$\sigma_\theta = \sigma_{\theta,p} \tag{8}$$

$$\sigma_z = \sigma_{z,p} + \sigma_{z,m} \tag{9}$$

Assuming the yield as limit, the admissible stress σ_{adm} is expressed by using Von Mises criterion for plane stress:

$$\sigma_{adm} = \sqrt{\sigma_z^2 - \sigma_z\sigma_\theta + \sigma_\theta^2} = \sigma_y \tag{10}$$

where σ_y is the yield stress.

Expressing σ_z as a function of σ_θ and σ_y Eq. (10) leads to Eq. (11):

$$\sigma_z = \frac{\sigma_\theta \pm \sqrt{4\sigma_y^2 - 3\sigma_\theta^2}}{2} \tag{11}$$

Introducing Eqs. (5) and (8) in (11) one has

$$\sigma_z = \frac{\frac{pD}{2t} \pm \sqrt{4\sigma_y^2 - 3\left(\frac{pD}{2t}\right)^2}}{2} \tag{12}$$

Eq. (12) represents the maximum admissible axial stress σ_z in a pipe with diameter D, thickness t, yield stress σ_y and internal pressure p before yield occurs. Just a part of σ_z calculated by Eq. (12) is available to resist moment, $\sigma_{z,m}$, since the pressure already results in the remaining part $\sigma_{z,p}$. Hence, combining Eqs. (6), (7), (9) and (12) one obtains the limiting load for yield in terms of maximum bending moment M_{max} as a function of pipe

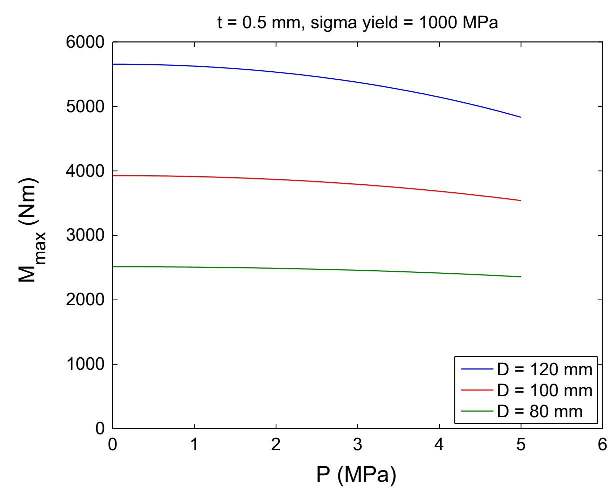

Fig. 1. Yield strength for different values of D.

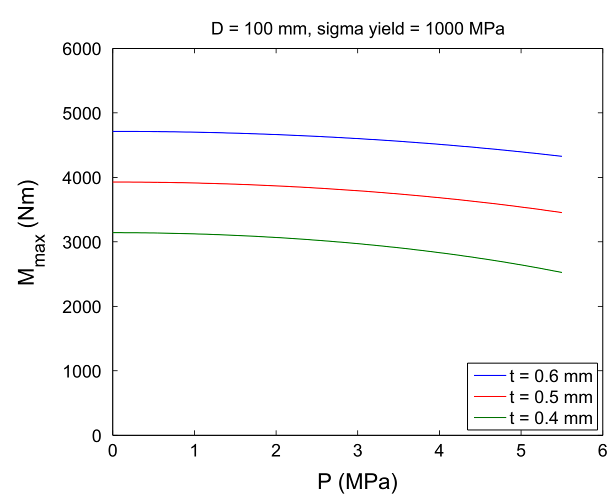

Fig. 2. Yield strength for different values of t.