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Review

Liquid-containment shells of revolution: A review of recent studies on strength, stability and dynamics

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ABSTRACT

In civil engineering, shell structures are widely used as liquid-containment vessels. Understanding how the shell responds to relevant loading conditions is important for the design of safe and economical liquid-containment shell structures. This paper reviews recent research on the strength, stability and vibration behaviour of liquid-containment shell structures, and traces the developments pertaining to the design of these facilities to withstand various loading and environmental effects such as liquid pressure, wind pressure, ground movement and thermal effects. Results of recent feasibility studies of non-conventional shell forms for liquid containment are also reported, and areas of focus for future research are suggested.

Contents

1. Introduction

Shells are widely used in the civil engineering industry for liquid containment [1,2]. Applications include elevated water tanks, storage vessels for the containment of petroleum products, liquefied gases and industrial chemicals, and water-treatment structures such as settling tanks and sludge digesters.

The vast majority of industrial metal tanks are of cylindrical (and to a lesser extent conical) shape owing to the ease of fabrication of shells of single curvature. Metal tanks of double curvature include spherical, ellipsoidal and toroidal vessels. For elevated water reservoirs, the mouldability of concrete into any desired shape has allowed these tanks to be constructed in a great variety of interesting shapes. In the 50 years or so, the same versatility of concrete gave rise to many architectural forms in the area of shell roof construction.

Thin shells have the advantages of high strength-to-weight ratio, functional effectiveness (excellent shape for containment), and good aesthetics. However, the property of thinness attracts special problems. One of the challenges is predicting how the shell

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responds to extreme loading and environmental conditions, which include liquid pressure (hydrostatic and hydrodynamic), wind pressure, sudden ground movements (earthquakes), impact, blast and temperature gradients. This understanding is vital for the design of safe and economical liquid-containment shell structures satisfying key performance requirements.

Over the past five decades, fundamental research on mathematical theories of shells has gradually given way to computational formulations such as the finite element method (FEM), the boundary element method (BEM) and the finite difference method (FDM), which are well suited to the study of complex shell problems. These methods have formed the basis of many studies of shell structures. Experimental methods have also played an important role, not only in their own right, but also as validation of the generally more economical numerical methods.

In particular, containment shell structures have been the subject of intense research over the last 50 years, and the literature in this area is substantial. Many review articles covering various aspects of shell structures have also appeared in the past. As far back as 1982, Tooth [3] surveyed storage vessels as an application of shells, while in 1996, Teng [4] reviewed the field of shell buckling, including tanks and silos.

This paper is based on a lecture that was presented at an international conference in 2012 [5]. We survey developments that have been reported in the literature since the turn of the millenium, with regard to a better understanding of the strength, stability and vibration behaviour of liquid-containment shell structures. These efforts have generally aimed at quantifying the relevant effects (critical buckling loads, natural frequencies, governing stresses, maximum deformations, etc), and proposing suitable design recommendations. It must be pointed out that there are many papers on the subject that have been published prior to 2000, but since the intention is to capture current trends in the field, it has been considered necessary to go back only as far as 2000; this period features a sufficiently large volume of literature to allow us to see the most significant trends. Reviews covering earlier contributions may be seen elsewhere in the literature [3,4].

Only shells of homogeneous construction are covered; laminated, sandwich and composite shells are outside the scope of this survey. Boilers and pressure vessels are also not included. The considerations in the earlier part of the paper mainly relate to metal shells, but studies reported in the later part of the paper are more relevant to construction in concrete. The aim of this review is not to look at every paper on liquid-containment shells that has appeared in the literature since 2000 (they are too many), but rather, to discuss the more representative of these studies, thus showing the recent areas of focus and general trends in research. Aspects of recent work of the author on the feasibility of new shell forms for liquid containment are also discussed. At the end, areas of focus for future research are suggested.

2. Buckling of vertical cylindrical tanks

Vertical cylindrical shells offer a convenient solution for the storage of water, petroleum products or chemicals, on account of the ease of manufacture of the cylindrical form (with its single curvature), the good containment properties of the cylindrical shape, and the structural efficiency of an axisymmetric distribution of primary loading (hydrostatic pressure). Problems associated with the buckling of the shell have been studied the most, given that the wall thickness t of these tanks is generally very small in relation to the radius r of the tanks. Metal tanks, where the r/t ratio typically lies in the range $500 \leq r/t \leq 2000$, are particularly vulnerable to buckling instability.

The buckling strength of a cylindrical shell subjected to an axial compressive load is a problem that has been studied many times, and interest on this topic still continues. With the development of design guidelines in mind, Kim and Kim found, in a study published in 2002 [6], that the buckling strength of such shells decreased significantly as the r/t ratio increases, while buckling strength decreased only slightly as the h/r (height-to-radius) ratio increased. A regression analysis of the numerical results led them to the formula:

$$\frac{\sigma_{cr}}{E} = 1.19\left(\frac{H}{D}\right)^{-0.0256}\frac{t}{D} \tag{1}$$

where σ_{cr} is the critical buckling stress, E the Young modulus, H/D the height-to-diameter ratio of the cylinder, and t/D the thickness-to-diameter ratio. For a thin steel shell with an H/D ratio of 1.0 and $t/D = 1/1000$, this would give a σ_{cr} value of 238 N/mm^2 (assuming an E value for steel of 200×10^9 N/m^2), which seems a little too high. Their formulation assumed that the cylinder is geometrically perfect.

The more general problem of the stability of circular cylindrical steel shells under simultaneous axial compression, torsion and external pressurisation (Fig. 1) was studied by Winterstetter and Schmidt [7]. In a paper published in 2002, they presented a proposal for the interactive buckling design of cylindrical steel shells on the basis of a comprehensive set of experimental and numerical results, for various types of analysis ranging from linear to fully nonlinear, as defined in Eurocode 3 Part 1–6 [8].

Cylindrical steel tanks with stepwise-varied wall thickness are a common form of construction. Membrane hoop stresses in the shell due to hydrostatic loading increase linearly with depth below the surface of the liquid, so it is reasonable to increment the thickness of the tank wall as one moves along the shell meridian

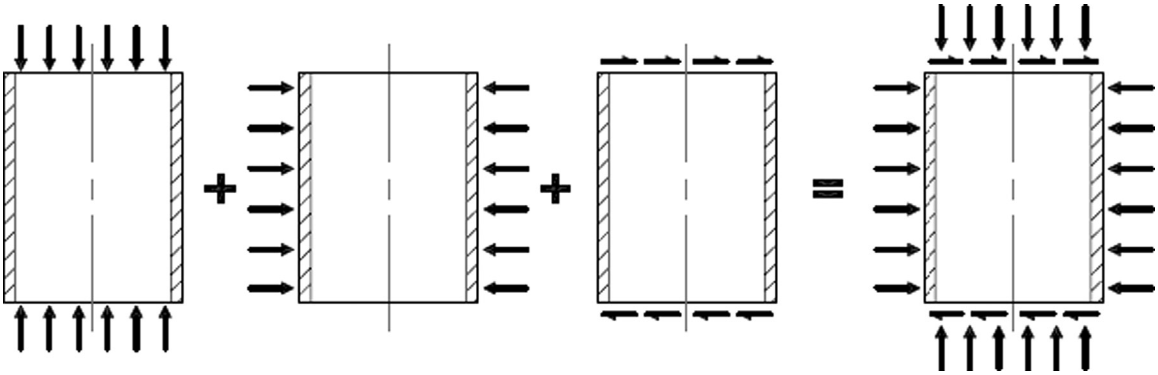

Fig. 1. Cylindrical shell under axial compression, external pressure, torsion and combined loading [7].